CHAPTER - 7
AREAS AND VOLUMES

7.0. General

One of the primary objects of most land surveys is to determine the area of the tract and volume of earth works. Areas are considered first of all, since the computation of areas is involved in the calculation of volumes, and are dealt with under the following headings:

(1) Mechanical Integration

(2) Areas enclosed by Irregular figures

(3) Areas enclosed by Regular figures

(4) Areas enclosed by straight lines

In ordinary land surveying, as discussed herein, the area of a tract of land is taken as its projection upon a horizontal plane, and it is not the actual area of the surface of the land. For precise determinations of the area of a large tract, such as a state or nation, the area is taken as the projection of the tract upon the earth's spheroidal surface at mean sea level.

7.1. Calculation of Areas

7.1.1. Mechanical Integration (The planimeter)

Mechanical integration by the planimeter can be applied to figures of all shapes, and although the construction and application of the instrument are simple (see Fig 7.1), the accuracy achieved is of the highest degree, particularly when measuring irregular figures.

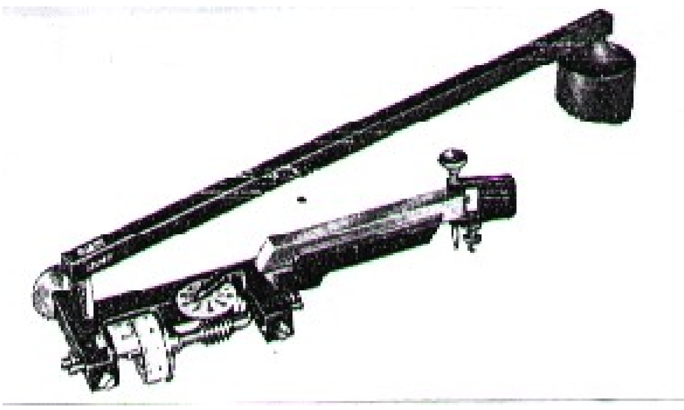


Fig 7.1 : Fixed index planimeter

It operation is easy and the procedure for measuring any area, the plan being on a flat horizontal surface, is –

(1) Place the pole outside the area in such a position that the tracing point can reach any part of the outline.

(2) With the tracing point placed on a known point on the outline, read the vernier.

(3) Move the tracing point clockwise around the outline, back to the known point, and read the vernier again.

(4) The difference between the two readings, multiplied by the scale factor, gives the area.

(5) Repeat until three consistent values are obtained, and the mean of these is taken.

Example 7.1: What is the area of a piece of land which has a plan area of 2.50 in.^{2} as measured by a fixed arm planimeter if the scale of the plan is (i) 1/2500; (ii) 6 in. to 1 mile. (iii) 1in. to 50ft.

Solution :#

(i) On this scale 1in. represents 2500 in.

$$1 \times 1 \text{ in.}^2 = 2500 \times 2500 \text{ in.}^2 = 6250000 \text{ in.}^2$$

$$1 \text{ in.}^2 = 6250000/(12\times12) = 6250000/144 = 43402.778 \text{ ft}^2$$

$$= 43402.778/(3\times 3) = 43402.778/9 = 4822.531 \text{ yd}^2$$

$$= 4822.531/4840 = 0.996 \text{ acres}$$

Thus the ground area = (plan area) x (scale factor)

$$= 2.50 \times 0.996 = \mathbf{2.49 \text{ acres}}$$

(ii) On this scale 6 in. = 1 mile

$$6 \times 6 \text{ in.}^2 = 1 \times 1 \text{ mile}^2 = 640 \text{ acres}$$

$$1 \text{ in.}^2 = 640/36 = 17.778 \text{ acres}$$

Thus the ground area = 2.50×17.778 = **44.44 acres**

(iii) On this scale 1 in. represents 50 ft.

$$1 \text{ in.} = 50 \text{ ft}$$

$$1 \times 1 \text{ in.}^2 = 50 \times 50 \text{ ft}^2 = 2500 \text{ ft}^2$$

$$1 \text{ in.}^2 = 2500/9 \text{ yd}^2 = 277.778 \text{ yd2}$$

$$= 277.778/4840 = 0.057 \text{ acres}$$

Thus the ground area = $(2.50) \times (0.057)$ = **0.1425 acres**

7.1.2. Areas enclosed by Irregular Figures

7.1.2.1. Trapezoidal Rule (Offsets at regular intervals)

Let Fig. 7.2 represent a position of a tract laying between a traverse line **AB** and irregular boundary **CD**, offsets $\mathbf{h_1, h_2, h_3, \ldots\ldots, h_n}$ having been taken at the regular intervals **d**. The summation of the areas of the trapezoids comprising the total area is:

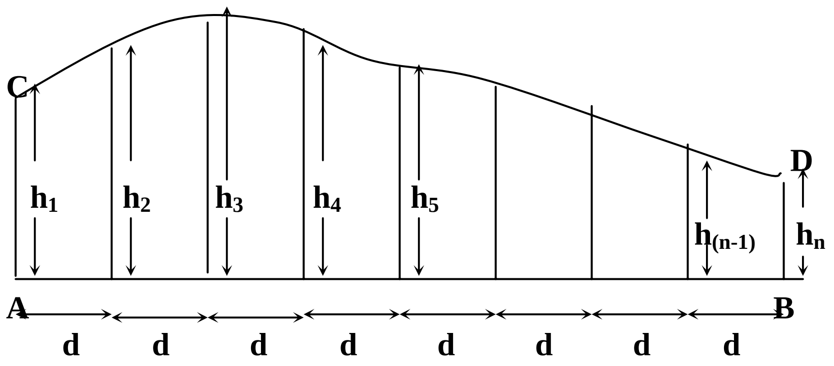


Fig 7.2

$$\text{Area of tract} = \frac{h_1 + h_2}{2} \cdot d + \frac{h_2 + h_3}{2} \cdot d + \cdots + \frac{h_{(n-1)} + h_n}{2} \cdot d$$

$$\textbf{Area of tract} = d \left[\frac{(h_1 + h_n)}{2} + h_2 + h_3 + \ldots\ldots\ldots + h_{(n-1)} \right] \qquad \ldots\ldots\ldots(7.1)$$

= d [(average of end offsets) + (sum of intermediate offsets) }

Trapezoidal Rule *: Add the average of the end offsets to the sum of the intermediate offsets. The product of the quantity thus determined the common interval between offsets is the required area.*

If offsets are taken sufficiently close together, the error involved in considering the boundary an straight between offsets is small as compared with the inaccuracies of the measured offsets.

Example7.2 : Calculate the area of the plot shown in Fig 7.3 if the offsets, scaled from the plan at regular intervals of 25 ft, are –

Offset	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
Length (ft)	55	60	53	48	54	68	65	57	55	50

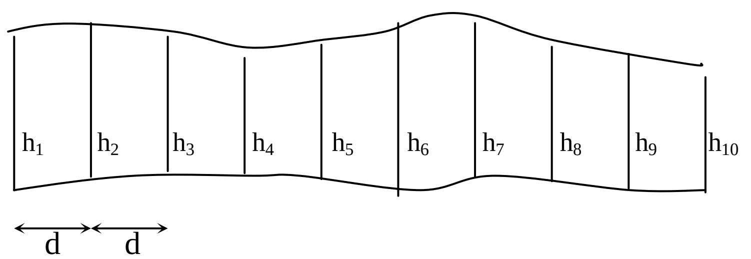


Fig 7.3

<u>Solution :#</u>

Area of tract == d [(average of end offsets) + (sum of intermediate offsets) }

$$= d \left[\frac{(h_1 + h_n)}{2} + (h_2 + h_3 + \ldots\ldots\ldots + h_{(n-1)}) \right]$$

$$\text{Area} = d\left[\frac{(h_1 + h_{10})}{2} + h_2 + h_3 + h_4 + h_5 + h_6 + h_7 + h_8 + h_9\right]$$

$$= 25[\,(55 + 50)/2 + 60 + 53 + 48 + 54 + 68 + 65 + 57 + 55\,]$$

$$= 25[\,(52.5) + (460)\,] \quad = \quad 25\,(512.5) \quad = \quad \mathbf{12812.5\ \ ft^2}$$

$$= 12812.5/9 = \mathbf{1423.611\ yd^2} = 1423.611/4840 \text{ acres} = \mathbf{0.294\ acres}$$

8.1.2.2. Simpson's One-third Rule (Offsets at regular interval)

In Fig 7.4 let **AB** be a portion of a traverse line**, DFC** a portion of the curved boundary to be arc of a parabola, and **h_1, h_2** and **h_3** any three consecutive rectangular offsets from traverse line to boundary taken at the regular interval **d.**

The area between traverse line and curve may be considered as composed of the trapezoid ABCD plus the area of the segment between the parabolic arc DFC and the corresponding chord DC.

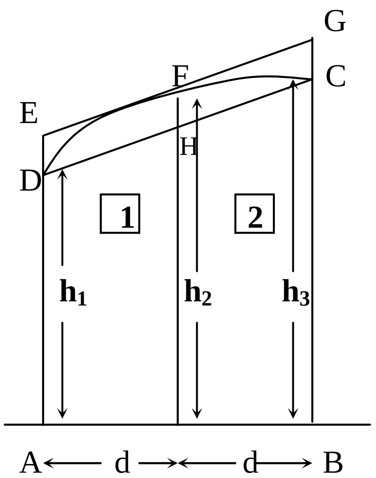


Fig 7.4

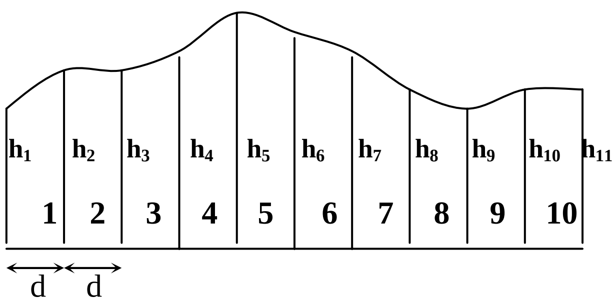


Fig 7.5

One property of a parabola is that the area of a segment (as DFC) is equal to two-third the area of the enclosing parallogram (as CDEFG). Then the area between the traverse line and curved boundary within the length of 2d (area contained between offset h_1 and h_3) is –

Area $_{1,2}$ = ABCFDA

= Trapezoid ABCHDA + Area CHDFC

$$= \frac{(h_1 + h_3)}{2}.2d + \frac{2}{3}\ (\text{ area of circumscribing parallelogram })$$

$$= \frac{(h_1 + h_3)}{2}\,2d + \frac{2}{3}\left(h_2 - \frac{(h_1 + h_3)}{2}\right)2d$$

$$= d/3\,(h_1 + 4h_2 + h_3)$$

Similarly for the next intervals (see Fig 7.5)

Area $_{3,4}$ = d/3 (h3 + 4h4 + h5)

The summation of these *partial areas for (n –1) intervals, n being an odd number* and representing the number of offsets , is –

$$\textbf{Area} = d/3\ [\ \{ h_1 + h_n\} + 2\{ h_3 + h_5 + h_7 + \ldots + h_{(n-2)}\} + 4\{ h_2 + h_4 + h_6 + \ldots + h_{(n-1)}\}\ ]$$

= d/3 [(sum of first and last offsets) + 2(sum of remaining odd offsets) + 4(sum of the even offsets)] ……(7.2)

This method, which gives greater accuracy than other methods, assumes that the irregular boundary is composed of a series of parabolic arcs. It is essential that the figure under consideration be divided into an even number of equal strips.

Simpson's One-third Rule ***:*** *Find the sum of the end offsets, plus twice the sum of the odd intermediate offsets, plus four times the sum of the even intermediate offsets. Multiply the quantity thus determine by one third of the common interval between offsets, and the result is the required area.*

Note : *(a) there must be an odd number of offsets.*

(b) The offsets must be at regular intervals, measured at right angles to the base line.

Example 7.3:In a survey the following offsets were taken to a fence from a traverse line(see Fig 7.5)

Chainage (ft)	0	20	40	60	80	100	120	140	160	180	200
Offset (ft)	55	60	58	62	70	65	63	58	54	57	56

Find the area between the fence and the traverse line in acres by the Simpson's One-third Rule.

Solution :#

d = (200 – 180) = (180 – 160) = ……….. = (40 – 20) = (20 – 0) = 20 ft

h_1=55, h_2= 60, h_3= 58, h_4= 62, h_5= 70, h_6= 65, h_7= 63, h_8= 58, h_9= 54, h_{10}= 57, h_{11}= 56

Area = d/3[(sum of first and last offsets) + 2(sum of remaining odd offsets) + 4(sum of even offsets)]

$= d/3[\ \{h_1 + h_n\} + 2\{h_3 + h_5 + h_7 + \ldots + h_{(n-2)}\} + 4\{h_2 + h_4 + h_6 + \ldots + h_{(n-1)})\}\ ]$

$= d/3[\ \{h_1 + h_n\} + 2\{h_3 + h_5 + h_7 + h_9\} + 4\{h_2 + h_4 + h_6 + h_8 + h_{10}\}\ ]$

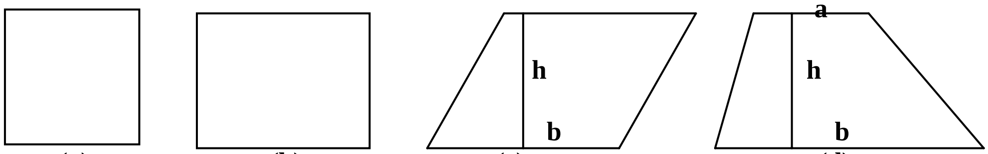
= 20/3[{55 + 56} + 2{58 + 70 + 63 + 54} + 4{60 + 62 + 65 + 58 + 57}]

= 20/3[{111} + 2{245} + 4{302}] = 20/3[{111} + {490} + {1208}]

= 20/3[1809] = **12060 ft^2** = 12060/9 yd^2 = **1340 yd^2**

= 1340/4840 acres **=** **0.2768 acres**

7.1.3. Areas Enclosed by Regular Figures

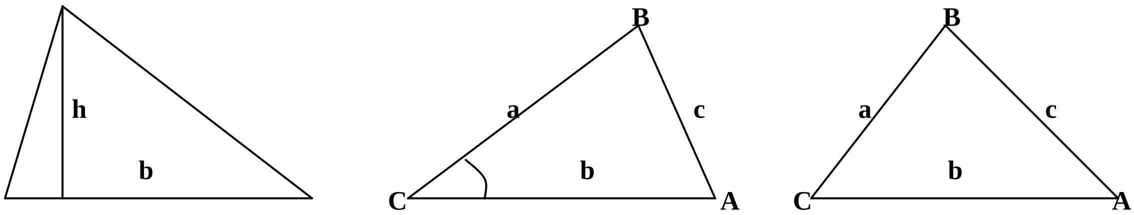


(a) **Square** Area = a^2

(b) **Rectangle** Area = ab

(c) **Parallelogram** Area = bh

(d) **Trapezoid** Area = [(a+b)/2] h

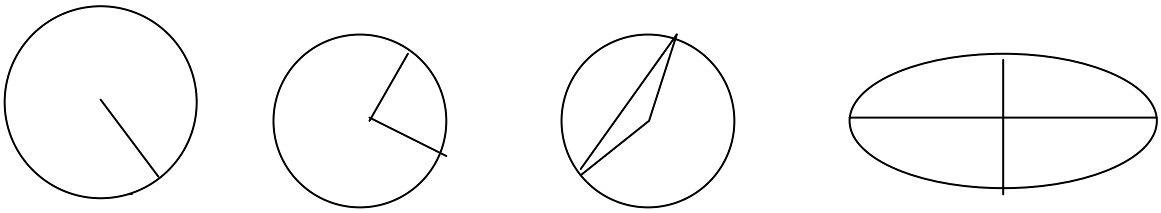


(e) **Triangles : Area = ½ bh**

(f) **Area = ½ ab Sin C**

(g) **Area =** $\sqrt{s(s-a)(s-b)(s-c)}$

s = ½ (a + b + c)



(h) **Circle** Area = $\Rightarrow r^2$

(i) **Sector** Area = $\Rightarrow r^2$(□/360)

(j) **Segment** Area = r^2/2 (□ rads - Sin□)

(k) **Elipse** Area = $\Rightarrow$ab

Fig 7.6

7.1.4. Areas Enclosed by Straight Line

7.1.4.1. Triangles Method

When the lengths of two sides and the included angle of any triangle are known, its area is given by the expression: (see Fig 7.6(f))

Area of triangle ABC = ½ ab Sin C = ½ bc Sin A = ½ ac Sin B …(7.3)

When the lengths of the three sides of any triangle are known, its area is determined by the equation.

Area of Triangle ABC = $\sqrt{s (s - a)(s - b)(s - c)}$ **…….(7.4)**

in which : s = ½ (a + b + c)

To use the equation7.3 and 7.4, straight boundary (polygon) is divided into possible and suitable triangles as Fig 7.7, 7.8 and 7.9 firstly and required lengths and angles are measured.

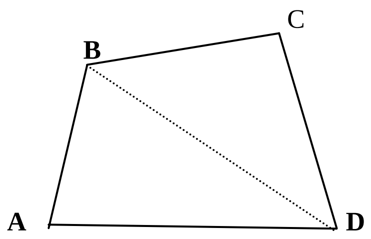


Fig 7.7

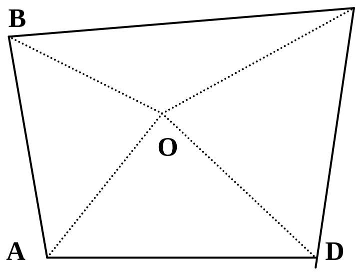


Fig 7.8

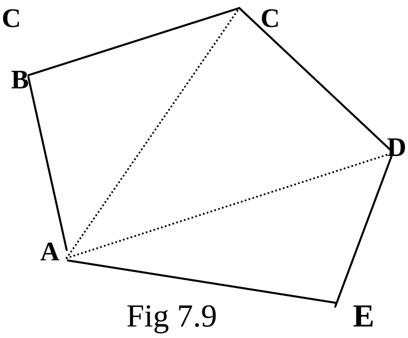


Fig 7.9

Example 7.4 : Determine the area of the tract ABCD. Measurement data are shown on the following figure 7.10.

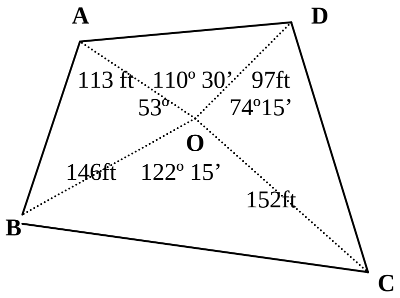


Fig 7. 10

<u>**Solution :#**</u>

Area of Triangle = ½ ab Sin C

Area of ABO = ½ (AO)(BO) Sin AOB

$= \frac{1}{2}(113)(146) \text{ Sin } 53^{\circ} = \frac{1}{2}(113)(146)(0.79863551) = 6587.944 \text{ ft}^2$

Area of ADO = ½ (AO)(DO) Sin AOD

$= \frac{1}{2}(113)(97) \text{ Sin } 110°30' = \frac{1}{2}(113)(146)(0.936672189 = 5133.432 \text{ ft}^2$

Area of CDO = ½ (DO)(CO) Sin COD

$= \frac{1}{2}(97)(152) \text{ Sin } 74°15\times = \frac{1}{2}(97)(152)(0.962455236) = 7095.22 \text{ ft}^2$

Area of BOC = ½ (BO)(CO) Sin BOC

$= \frac{1}{2}(146)(152) \text{ Sin } 122°15' = \frac{1}{2}(146)(152)(0.845727821) = 9384.196 \text{ ft}^2$

Area of ABCD = Area of ABO + Area of ADO + Area of CDO + Area of BOC

= 6587.944 + 5133.432 + 7095.22 + 9384.196

= 28200.792 $\mathbf{ft^2}$ = 28200.792/9 = 3133.421333 $\mathbf{yd^2}$

= 3133.42133/4840 = 0.647401101 = 0.647 **acres**

7.1.4.2. Coordinate Method

When the coordinates of the corner of track are known, the area of tract is determined by following.

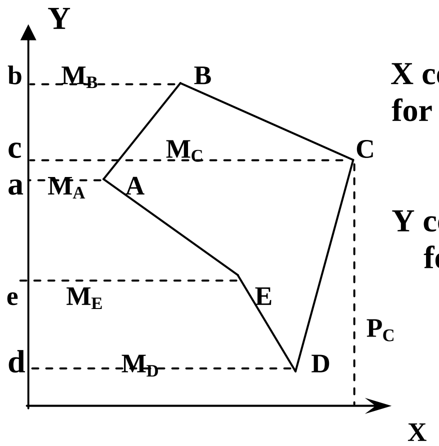


Fig 7. 11

X coordinate = Total departure = Meridian = Eastings for point A

$$\mathbf{X_A = \quad = M_A = E_A}$$

Y coordinate = Total Latitude = Parallel = Northing for point A

$$\mathbf{Y_A = \quad = P_A = N_A}$$

Area ABCDEA = area BCcb + area CDdc – area DEcd – area EAac – area ABba

$= ½ (M_B + M_C)(P_B – P_C) + ½ (M_C + M_D)(P_C – P_D) – ½ (M_D + M_E)(P_E – P_D)$

$- ½ (M_E + M_A)(P_A – P_E) – ½ (M_A + M_B)(P_B – P_A)$

2 Area ABCDEA=$(P_AM_B+P_BM_C+P_CM_D+P_DM_E+P_EM_A)–(M_AP_B+M_BP_C+M_CP_D+M_DP_E+ M_EP_A)$

$$= \begin{array}{cccccc} M_A - & M_B - & M_C - & M_D - & M_E - & M_A \\ ---- & ---- & ---- & ---- & ---- & ---- \\ P_A + & P_B + & P_C + & P_D + & P_E + & P_A \end{array}$$

$$\textbf{Area ABCDEA} = ½ \left[\begin{array}{cccccc} \mathbf{M_A} - & \mathbf{M_B} - & \mathbf{M_C} - & \mathbf{M_D} - & \mathbf{M_E} - & \mathbf{M_A} \\ ---- & ---- & ---- & ---- & ---- & ---- \\ \mathbf{P_A} + & \mathbf{P_B} + & \mathbf{P_C} + & \mathbf{P_D} + & \mathbf{P_E} + & \mathbf{P_A} \end{array}\right] \quad \textbf{….(7.5)}$$

Coordinate method : *The difference between the sum of the products of the coordinates joined by the downward arrow lines and the sum of the products of the coordinates joined by the upward arrow lines is equal to twice the area of the tract.*

Example 7.5 : Find the area of the tract by coordinate method. Data are tabulated as follow :

Points	A	B	C	D	E
X Coordinate(m)	1000.0	786.4	612.0	701.2	811.9
Y Coordinate(m)	1000.0	801.3	982.4	1316.5	1514.4

Solution :#

$$\text{Area ABCDEA} = ½ \left(\begin{array}{cccccc} X_A - & X_B - & X_C - & X_D - & X_E - & X_A \\ ---- & ---- & ---- & ---- & ---- & ---- \\ Y_A + & Y_B + & Y_C + & Y_D + & Y_E + & Y_A \end{array}\right)$$

Area ABCDEA=½$[(Y_AX_B+Y_BX_C+Y_CX_D+Y_DX_E+Y_EX_A)–(X_AY_B+X_BY_C+X_CY+ X_DY_E+X_EY_A)]$

$$= \tfrac{1}{2} [\{(1000.0)(786.4) + (801.3)(612.0) + (982.4)(701.2) + (1316.5)(811.9) + (1514.4)(1000.)\} - \{(1000.0)(801.3)+(786.4)(982.4)+ (612.0)(1316.5) + (701.2)(1514.4) + (811.9)(1000.0)\}]$$

$$= \tfrac{1}{2} [4575920.8 - 4272287]$$

$$= \tfrac{1}{2} [303633.79]$$

Area ABCDEA = **151816.9 m^2**

7.1.4.3. Double Meridian Distance (D.M.D) Method

The meridian distance of a point is the total departure or perpendicular distance from the reference meridian; thus in Fig 7. 12 the meridian distance of **B** is **Bb** and is positive. The meridian distance of a straight line is the meridian distance of its midpoint. The double meridian distance of a straight line is the sum of the meridian distances of the two extremities; thus double meridian.

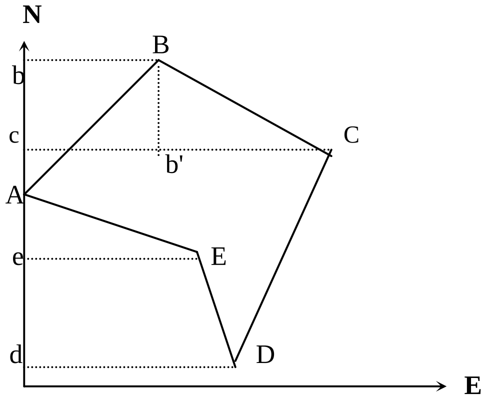


Fig 7.12

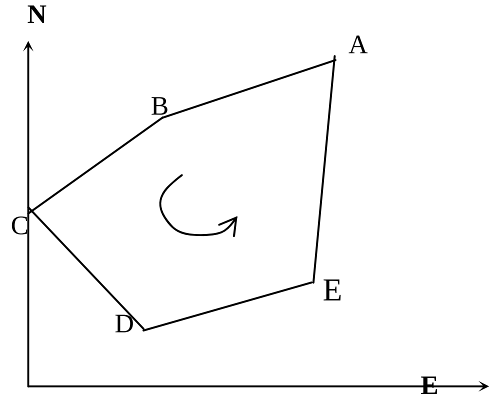


Fig 7.13

distance of **BC** is **Bb + Cc**. It is clear that if the meridian passes through the most westerly corner of the traverse, the double meridian distance of all line will be positive, which is a convenience in computing. The length of the orthographic projection of a line upon the meridian is the latitude of the line.

Double Area = D.M.D x Lat. ………….. (7.6)

Hence the sign of a double area is the same as that of the corresponding Lat. Thus in Fig 7.12 the double **area of AbB, DdeE** and **EeA** are **positive**, the **Lat. Ab, de** and **eA** being **positive**, while the double **area of CcbB** and **DdcC** are **negative**, the **Lat. bc** and **cd** being **negative**. Since the projected areas outside the traverse are considered once as positive and once as negative, the algebraic sum of their double area is zero. Therefore the algebraic sum of all double areas is equal to twice the area of the tract within the traverse.

Where this algebraic sum of the double areas is a positive or negative according the order the lines of the traverse are considered a clockwise order of lines, result in a negative double area, and a counterclockwise order result in a positive double area. The sign of the area is not significant.

Following are three covenient rules for **determining D.M.D's** which are deduced from the relations just illustrated.

(1) The D.M.D of the first course (renocked from the point through which the reference meridian passes) is equal to the departure (Dep.) of that course.

(2) The D.M.D of any other courses is equal to the D.M.D of the preceding course, plus the departure(Dep.) of the preceding course, plus the departure (Dep.) of the course itself.

(3) The D.M.D of the last course is numerically equal to the departure(Dep.) of the course but that opposite sign.

The first two rules are employed in computing values. The third rule is useful as a check on the correctness of the computation.

Example7.6 : Find the area of the closed traverse by D.M.D method. Corrected Lat. and Dep. are tabulated as follow :

Line	Corrected Lat.	Corrected Dep.
AB	- 5.693	- 33.990
BC	- 21.361	- 13.911
CD	- 28.201	+ 18.867
DE	+ 1.013	+ 28.608
EA	+ 54.242	+ 0.426

Solution :#

Computation for Area of Traverse by Double Meridian Distances Method (Draw as Fig 7.13)

Line	D.M.D	Latitude	Double Area +	Double Area -
CD	D.M.D = 18.867	- 28.201		532.06
	Dep. CD = + 18.867			
	Dep. DE = + 28.608			
DE	D.M.D = + 66.342	+ 1.013	67.21	
	Dep. DE = + 28.608			
	Dep. EA = + 0.426			
EA	D.M.D = + 95.376	+ 54.242	5173.51	

	Dep. EA = + 0.426			
	= + 95.802			
	Dep. AB = - 33.990			
AB	D.M.D = + 61.812	- 5.693		
	Dep. AB = - 33.990			351.89
	= + 27.822			
	Dep. BC = - 13.911			
BC	D.M.D = + 13.911	- 21.361		297.15
			5240.72	1181.1
			- 1181.1	
			+ 4059.62	

Double Area = 4059.62

Area = 4059.62/2 = 2029.81 m^2

7.1.4.4. Double Parallel Distances (D.P.D.) Method

Determining area within a closed traverse by the method of double parallel distances (D.P.D's) is essentially the same as the D.M.D method described in preceding articles. The only difference is that the courses are projected upon a parallel instead of upon the meridian.

Although the D.P.D. method, possesses all the advantages of the D.M.D method. It is used very little in practice. It is occasionally employed as an independent method of checking areas, which have been computed by the D.M.D method.

7.2. Units of Area

	in^2	**ft^2**	**yd^2**	**Acres**	**$mile^2$**	**mm^2**	**cm^2**	**m^2**	**km^2**	**Hectares**
in^2	1	$6.944x10^{-3}$	$7.716x10^{-4}$	$1.594x10^{-7}$	$2.491x10^{-10}$	$6.452x10^{2}$	6.452	$6.452x10^{-4}$	$6.452x10^{-10}$	$6.452x10^{-8}$
ft^2	144	1	$0.111x10^{-1}$	$2.296x10^{-5}$	$3.587x10^{-8}$	$9.29x10^{4}$	$9.29x10^{2}$	$9.29x10^{-2}$	$9.29x10^{-8}$	$9.29x10^{-6}$
yd^2	1296	9	1	$2.066x10^{-4}$	$3.228x10^{-7}$	$8.361x10^{5}$	$8.361x10^{3}$	$8.361x10^{-1}$	$8.361x10^{-7}$	$8.361x10^{-5}$
Acre	6272640	43560	4840	1	$1.563x10^{-3}$	$4.047x10^{9}$	$4.047x10^{7}$	$4.047x10^{3}$	$4.047x10^{-3}$	$0.047x10^{-1}$
$mile^2$	$4.014x10^{9}$	27878400	3097600	640	1	$2.590x10^{12}$	$2.590x10^{10}$	$2.590x10^{6}$	2.590	$2.590x10^{2}$
mm^2	$1.55x10^{-3}$	$1.076x10^{-5}$	$1.196x10^{-6}$	$2.471x10^{-10}$	$3.861x10^{-13}$	1	$1.0x10^{-2}$	$1.0x10^{-6}$	$1.0x10^{-12}$	$1.0x10^{-10}$
cm^2	$1.55x10^{-1}$	$1.076x10^{-3}$	$1.196x10^{-4}$	$2.471x10^{-8}$	$3.861x10^{-11}$	100	1	$1.0x10^{-4}$	$1.0x10^{-10}$	$1.0x10^{-8}$
m^2	$1.55x10^{3}$	$1.076x10^{1}$	1.196	$2.471x10^{-4}$	$3.861x10^{-7}$	$1.0x10^{6}$	$1.0x10^{4}$	1	$1.0x10^{-6}$	$1.0x10^{-4}$
km^2	$1.55x10^{9}$	$1.076x10^{7}$	$1.196x10^{6}$	$2.471x10^{2}$	$3.861x10^{-1}$	$1.0x10^{12}$	$1.0x10^{10}$	$1.0x10^{6}$	1	$1.0x10^{2}$
hectare	$1.55x10^{7}$	$1.076x10^{5}$	11959.64	2.741	$3.861x10^{-3}$	$1.0x10^{10}$	$1.0x10^{8}$	$1.0x10^{4}$	$1.0x10^{-2}$	1

7.3. Calculation of Volumes (Earthwork)

7.3.1. General

Earthwork operations involved the determination of the volumes of material which must be excavated or embanked on an engineering project to bring the ground surface to a predetermined grade, and the setting of stakes to aid in carrying out the construction work according to the plans. Although the term earthwork is used, the principles involved in determining volumes apply equally well to volumes of concrete structures, to volumes of stock piles of crushed stone, gravel, sand, coal and ore, and to volumes of reservoirs, mine. The fieldwork includes the measurements of the dimensions of the various geometrical solid which make up the volumes, the setting of grade stakes, and the keeping of the field notes. The office work involved the computations of the measured volumes and the determination of the most economical manner of performing the work.

In general, the earthworks fall into :

(1) Long narrow earth of varying depths (roadway cutting and embankments)

(2) Wide flat earthworks (reservoirs, sports pitches, car parks, etc.)

There are three general methods for calculating earthworks :

(I) By cross sections

(II) By contours

(III) By spot heights

7.3.2. Volumes from Cross Sections

In this method cross sections are taken at right angles to some convenient line which runs longitudinally through the earthworks and although it is capable of general application it is probably most used on long narrow works such as roads, railways, canals, embankments, pipe excavations, etc. The volume of earthwork between successive cross sections are calculated from a consideration of the cross-sectional areas, which in turn are measured or calculated by the general methods already given, i.e. by planimeter, division into triangles, coordinates, etc.

A cross section is a section taken normal to the direction of the proposed center line of an engineering project, such as a highway, railroad, trench, earth dam, or canal. A cross section for these would have similar characteristic. It is bounded by a **base (formation)** , **side slopes**, and the **natural terrain** (see Fig7.14). The inclination of a side slope is defined by the horizontal distance **m** on the slope corresponding to **a unit vertical distance.** The slope may be a raise (in excavation) or a fall (in embankment). A side slope of 3 to 1, for example, means that for each 3 ft of horizontal distance the side slope rise or fall 1 ft. This can be designated as 3 : 1 or 1 on 3.

In long constructions which have constant formation width and side slopes, however, it is possible to simplify the computation of cross-sectional areas by the used of formulae. There are specially useful for railways, long embankments, etc., and formula will be given for the following types of cross section :

(a) sections level across

(b) sections with a cross fall

(c) sections part in cut and part in fill

(d) sections of variable levels

7.3.2.1. Sections Level Cross

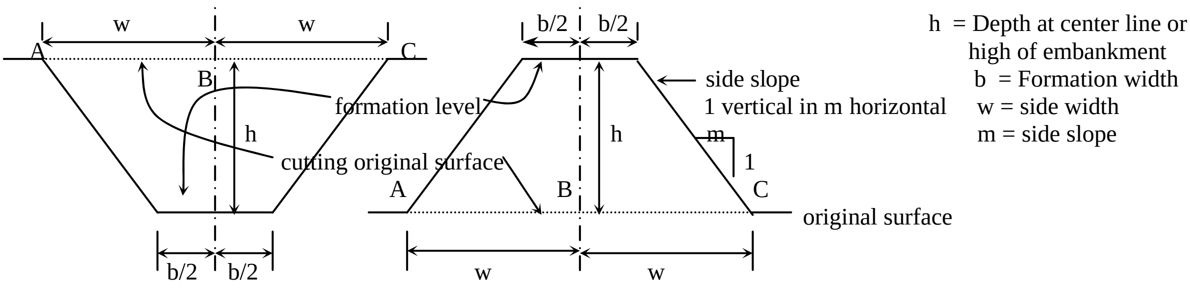


Fig 7.14 : Cutting and Embankment (Section Level Cross)

Side width = w = b/2 + mh …………..(7.7)

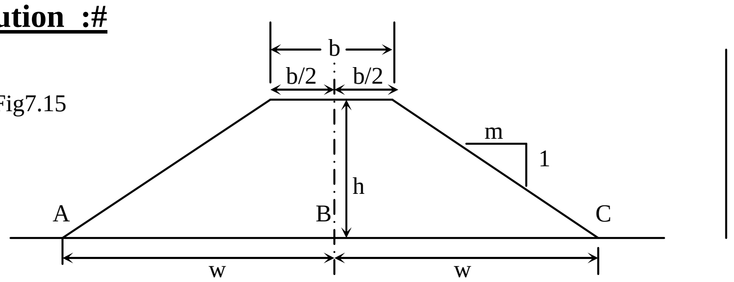
Cross-sectional Area = A = h (b + mh) …………..(7.8)

In setting out this section, for soil stripping and fixing the toe of each side slope **m**, pegs would be intserted at A and C at distances of **w** from center peg B measured normal to the center line.

<u>Example 7.7 :</u> At a certain station an embankment formed on level ground has a height at its center line of 12 ft. If the formation width is 40 ft, find (a) the side widths, and (b) the area of cross section area, given that the side slope is 1 vertical to 2 ½ horizontal.

<u>Solution :#</u>

Fig7.15



h = 12 ft

b = 40 ft

m = 2 ½ = 2.5

side width = w = ?

cross-sectional area = A = ?

(a) Side widths are both equal = w = b/2 + mh

$$= 40/2 + 2.5 \times 12 = 20 + 30$$

$$= \mathbf{50\ ft}$$

(b) Cross-sectional Area $= A = h(b + mh)$

$$= 12(40 + 2.5 \times 12) = 12(40 + 30) = 12(70)$$

$$= \mathbf{840\ ft^2}$$

In setting out this section, for soil stripping and fixing the toe of each side slope, pegs would be inserted at A and C at distances of 50 ft from center peg B measured normal to the center line.

7.3.2.2. Sections with Cross Fall

In this case the existing ground has cross fall or transverse gradient relative to the center line, and the side widths are not equal since the section is not symmetrical about the center line.

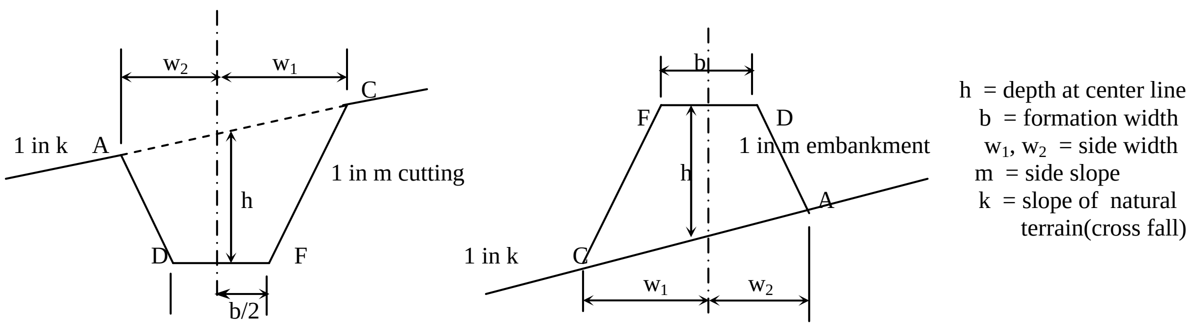


Fig 7.16 . Cutting and Embankment (Section with Cross Fall)

$$\textbf{Side Width} = w_1 = \left(\frac{b}{2} + mh\right)\left(\frac{k}{k - m}\right) \qquad \ldots\ldots\ldots\ldots(7.9)$$

$$\textbf{Side Width} = w_2 = \left(\frac{b}{2} + mh\right)\left(\frac{k}{k + m}\right) \qquad \ldots\ldots\ldots\ldots(7.10)$$

The Area of the cutting or the embankment is the area ACFDA,

$$\textbf{Cross-section Area} = A = \frac{1}{2m}\left[\left(\frac{b}{2} + mh\right)(w_1 + w_2) - \left(\frac{b^2}{2}\right)\right] \qquad \ldots\ldots(7.11)$$

This type of section is sometimes known as a **" two level section "**, since two levels are required to establish the cross fall of **1 in k**.

Example 7.8 : Calculate the side widths and cross-sectional area of an embankment to a road with formation width of 40 ft, and side slopes 1 vertical to 2 horizontal, when the center height is 10 ft and the existing ground has a cross fall of 1 in 12 at right angle to the center line of the embankment.

Solution :#

Referring to Fig 7.16

Where $b = 40$ ft, $m = 2$, $h = 10$ ft , $k = 12$

$$\text{Side Width} = w_1 = \left(\frac{b}{2} + mh\right)\left(\frac{k}{k-m}\right)$$

$$= (40/2 + 2 \times 10)(12/(12-2)) = (20 + 20)(12/10)$$

$$= (40)(1.2) = \mathbf{48\ ft}$$

$$\text{Side width} = w_2 = \left(\frac{b}{2} + mh\right)\left(\frac{k}{k+m}\right)$$

$$= (40/2 + 2 \times 10)(12/(12+2)) = (20 + 20)(12/14)$$

$$= (40)(12/14) = \mathbf{34.29\ ft}$$

$$\text{Cross-sectional Area} = A = \frac{1}{2m}\left[\left(\frac{b}{2} + mh\right)(w_1 + w_2) - \frac{b^2}{2}\right]$$

$$= 1/2(2)\ [\ (40/2 + 2 \times 10)(48 + 34.29) - (40)(40)/2\ ]$$

$$= 1/4\ [(20 + 20)(82.29) - 1600/2] = 1/4\ [(40)(82.29) - 800]$$

$$= 1/4\ [3291.6 - 800] = 1/4\ [2491.6] = \mathbf{622.9\ ft^2}$$

7.3.2.3. Section Part in Cut and Part in Fill

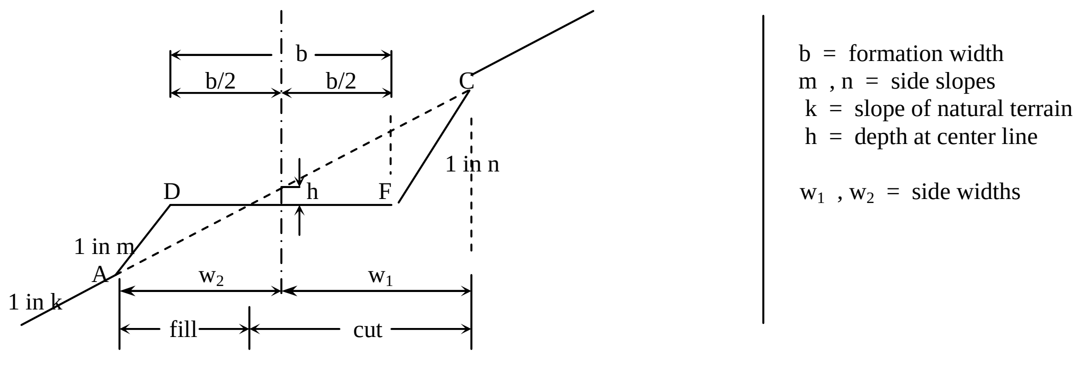


Fig 7.17 : Section part in Cut and Part in Fill

$$\textbf{Side Width} = w_1 = \left(\frac{k}{k-n}\right)\left(\frac{b}{2} + nh\right) \qquad \text{……………..} (7.12)$$

$$\textbf{Side Width} = w_2 = \left(\frac{k}{k-m}\right)\left(\frac{b}{2} - mh\right) \qquad \text{……………..} (7.13)$$

$$\textbf{Area of Fill} = \frac{1}{2}\left(\frac{(b/2 - kh)^2}{(k-m)}\right) \qquad \text{……………...} (7.14)$$

$$\textbf{Area of Cut} = \frac{1}{2}\left(\frac{(b/2 + kh)^2}{(k - n)}\right) \qquad \text{……………… (7.15)}$$

When the cross section is in **fill at the center line**, instead of being in cut as in Fig 7.17, then the following modified formula obtain :

$$\textbf{Side Width} = w_1 = \left(\frac{k}{k - n}\right)\left(\frac{b}{2} - nh\right) \qquad \text{…………….. (7.12a)}$$

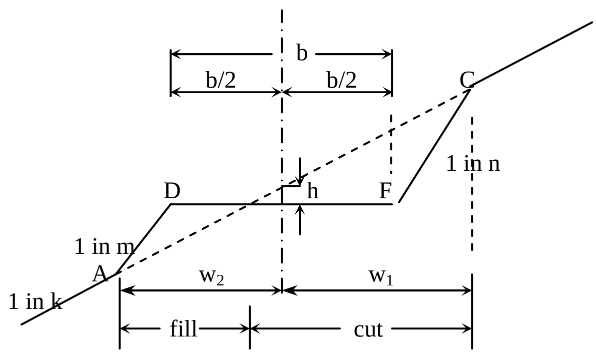
$$\textbf{Side Width} = w_2 = \left(\frac{k}{k - m}\right)\left(\frac{b}{2} + mh\right) \qquad \text{…………….. (7.13a)}$$

$$\textbf{Area of Fill} = \frac{1}{2}\left(\frac{(b/2 + kh)^2}{(k - m)}\right) \qquad \text{……………… (7.14a)}$$

$$\textbf{Area of Cut} = \frac{1}{2}\left(\frac{(b/2 - kh)^2}{(k - n)}\right) \qquad \text{……………… (7.15a)}$$

Example 7.9 : A road has a formation width of 15m and side slope of 1 vertical to 1 horizontal in cut and 1 vertical to 2 horizontal in fill. The original ground has a cross fall of 1 vertical to 5 horizontal. If the depth of excavation at the center line is 1m. Calculate the side widths and areas of cut and fill.

<u>Solution :#</u>



$b = 15$m
$m = 2,\ n = 1$
$k = 5$
$h = 1$m

$w_1,\ w_2 = ?$

Area of cut = ?
Area of fill = ?

$$\text{Side Width} = w_1 = \left(\frac{k}{k - n}\right)\left(\frac{b}{2} + nh\right)$$

$$= (5/(5-1))\,(15/2 + 1 \times 1) = (5/4)\,(7.5 + 1)$$

$$= (5/4)\,(8.5) = 10.625 \text{ m}$$

$$\text{Side Width} = w_2 = \left(\frac{k}{k - m}\right)\left(\frac{b}{2} - mh\right)$$

$$= (5/(5-2)\ (15/2 - 2 \times 1) = (5/3)\,(7.5 - 2)$$

$$= (5/3)\,(5.5) = 9.167 \text{ m}$$

$$\text{Area of Fill} = \frac{1}{2}\left(\frac{(b/2 - kh)^2}{(k - m)}\right)$$

$$= (1/2)\,[\,(15/2 - 5 \times 1)^2 / (5 - 2)\,] = (1/2)\,[\,(7.5-5)^2 / (3)\,]$$

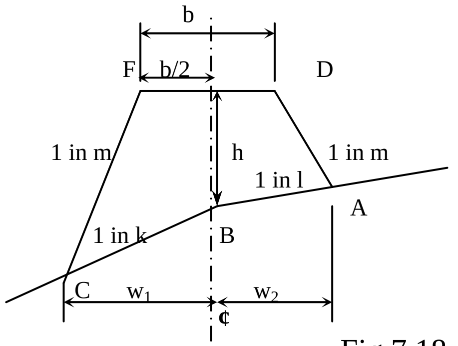
$$= (1/2)\,[\,(2.5)^2 / 3\,] = (1/2)\,(\,(6.5)/3\,) = 1.042\ m^2$$

$$\text{Area of Cut} = \frac{1}{2}\left(\frac{(b/2 + kh)^2}{(k - n)}\right)$$

$$= (1/2)\,[\,(15/2 + 5 \times 1)^2 / (5 - 1)\,] = (1/2)\,[\,(7.5 + 5)^2 / (4)\,]$$

$$= (1/2)\,[\,(12.5)^2 / (4)\,] = (1/2)\,[\,(156.25 / 4\,] = 19.531\ m^2$$

7.3.2.4. Section of Variable Level



h = depth of center line
b = formation width
m = side slope
k, l = slopes of natural terrain
w_1, w_2 = side widths

Fig 7.18 : Embankment (Three Level Section)

The type of section shown in Fig 7.18 is sometimes referred to as a **" three level section "**
since at least three levels are required on each cross section to enable the ground slopes to be calculated. The side width formula are obtained as for two level sections, and are :

$$\textbf{Side Width} = w_1 = \left(\frac{b}{2} + mh\right)\left(\frac{k}{k - m}\right) \quad \ldots\ldots\ldots\ldots\ldots\ (7.16)$$

$$\textbf{Side Width} = w_2 = \left(\frac{b}{2} + mh\right)\left(\frac{l}{l + m}\right) \quad \ldots\ldots\ldots\ldots\ldots\ (7.17)$$

If **BA** were falling away from the center line,

$$\textbf{Side Width} = w_2 = \left(\frac{b}{2} + mh\right)\left(\frac{l}{l - m}\right) \quad \ldots\ldots\ldots\ldots\ (7.17a)$$

$$\textbf{Cross-sectional Area} = A = \frac{1}{2m}\left[(w_1 + w_2)\left(mh + \frac{b}{2}\right) - \frac{b^2}{2}\right] \quad \ldots\ldots\ (7.18)$$

An irregular cross section is illustrated in Fig 7.19. The area of this type of section can be determined by dividing the section into trapezoids and triangle. However, the most efficient procedure is to use the coordinates method as developed to determined the area within a closed traverse in Equation 7.5**.** It is essential that, plotting of cross sections with suitable scale. And the cross sectional areas also can be found by the planimeter.

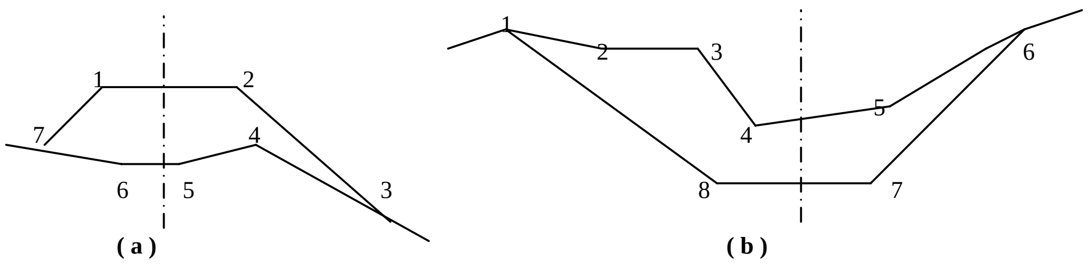


Fig 7. 19 : Irregular Cross Section

7.4. Computation of Volume

Having determined the various areas of cross section, the volume of earth involved in the construction are computed by one of the following methods : (1) mean areas, (2) end areas, (3) prismoidal formula.

7.4.1. Volumes by Means Areas

In this method the volume is determined by multiplying the mean of the cross-sectional areas by the distance between the end sections. If the areas are $\mathbf{A_1, A_2, A_3, A_4,\ldots,A_{n-1}, A_n}$, and the distance between the two extreme sections $\mathbf{A_1}$ and $\mathbf{A_n}$ is **L**, then

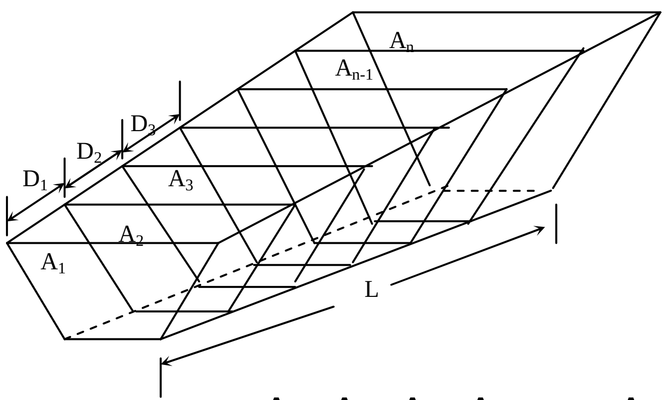


Fig 7. 20

$$\textbf{Volume} = V = \left(\frac{A_1 + A_2 + A_3 + A_4 + \ldots\ldots + A_{n-1} + A_n}{n} \right) L \qquad \ldots\ldots\ldots (7.19)$$

The method is not a very accurate one.

7.4.2. Volumes by End Areas

If $\mathbf{A_1}$ and $\mathbf{A_2}$ are the areas of two cross sections distance **D** apart (see Fig 7.20), then the volume V between the two is given by :

$$\textbf{Volume} = V = \left(\frac{A_1 + A_2}{2}\right) D \qquad \text{……………..} (7.20)$$

This expression is correct so long as the area of the section mid-way between A1 and A2 is the mean of the two, and such can be assumed to be the case so long as there is no wide variation between successive sections.

For a series of consecutive cross section (see Fig 7.20), the total volume will be :

$$\textbf{Volume} = V = \left(\frac{A_1+A_2}{2}\right) D_1 + \left(\frac{A_2+A_3}{2}\right) D_2 + \left(\frac{A_3+A_4}{2}\right) D_3 + \ldots \qquad .. (7.21)$$

If $D_1 = D_2 = D_3 = \ldots\ldots = D$

$$\textbf{Volume} = V = D\left[\left(\frac{A_1 + A_n}{2}\right) + (A_2 + A_3 + A_4 + \ldots\ldots + A_{n-1})\right]$$

= D [(average of end areas) + (sum of intermediate areas)] . (7. 22)

Which is sometimes referred to as the **Trapezoidal Rule** for volumes.

Example 7.10 : A Cutting is formed on ground which is level traverse to the cutting but falling at 1 in 20 longitudinally so that three section 100 ft apart have center line depth of 20 ft, 25 ft, and 30 ft respectively below original ground level. If side slope of 1 in 1 are used, determine the volume of cut between the outer sections when the formation width is 20 ft using the trapezoidal rule, end area method use only extreme sections.

<u>**Solution :#**</u>

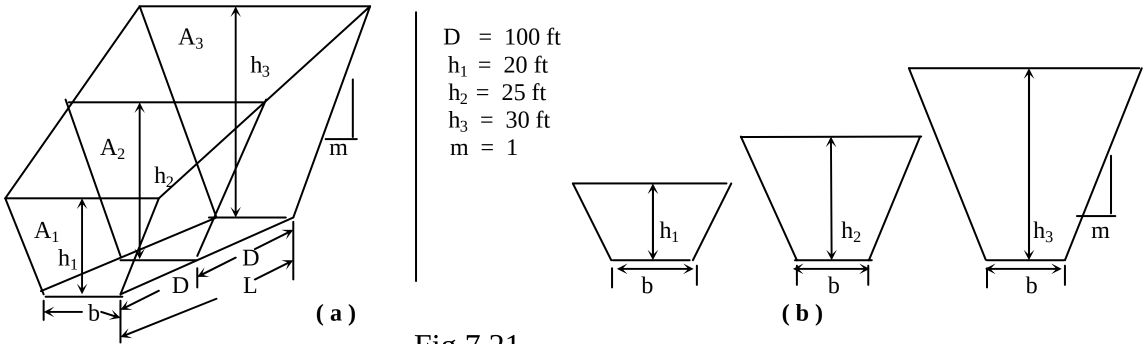


Fig 7.21

<u>For Section Level Cross</u>

Cross-sectional Area = A = h (b + mh)

A_1= 20 (20 + 1 x 20) = 20(20 + 20) = 20 (40) = 800 ft^2

A_2= 25 (20 + 1 x 25) = 25(20 + 25) = 25 (45) = 1125 ft^2

A_3= 30 (20 + 1 x 30) = 30(20 + 30) = 30 (50) = 1500 ft^2

Note that the mid-area A_2 not the mean of ($A_1 + A_2$)

By Trapezoidal Rule

$$\text{Volume} = V = D\,[\,(\text{average of end areas}) + (\text{sum of intermediate areas})\,]$$

$$= D\left[\left(\frac{A_1 + A_3}{2}\right) + A_2\right]$$

$$= 100\,[\,(800 + 1500)/2 \;+\; 1125\,] \;=\; 100[\,(2300)/2 \;+\; 1125\,]$$

$$= 100\,[\,1150 + 1125\,] \;=\; 100\,[\,2275\,] \;=\; 227500\ ft^3$$

$$= 227500/27 \;=\; 8426\ yd^3$$

By End Area Method (only extreme sections)

$$\text{Volume} = V = D\left[\frac{A1 + A3}{2}\right] \quad \text{, where } D = L = 200\ ft$$

$$= 200\,[(800 + 1500)/2] = 200\,[2300/2] = 200\,[1150] = 23000\ ft^3$$

$$= 23000/27 \;=\; 8519\ yd^3$$

7.4.3. Volume by Prismoidal Formula

If the volume of earth between two successive cross sections be considered a **prismoid** then a more precise formula (the prismoidal formula) may be used. It is generally considered that, all thing being equal, use of this formula gives the most accurate estimate of volume.

A prismoid is defined as any solid having two plane parallel faces regular or irregular in shape, which can be joined by surfaces either plane or curved on which a straight line may be from one of the parallel ends to the other. i.e sides, top and bottom, must be formed by straight continuous lines running from one end face to other.

$$\textbf{Volume} = D/6\,(A_1 + 4M + A_2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(7.23)$$

Where $\mathbf{A_1}$ and **A2** are the areas of the two end faces distances **D** apart, **M** is the area of the section mid-way between.

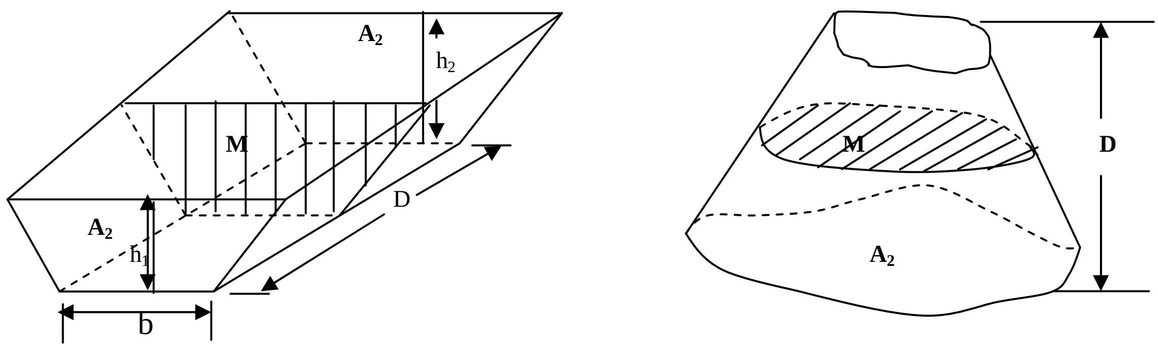


Fig 7.22

Note :*The area of **M** is **NOT** the mean areas **A1** and **A2**. The area must be calculated from its own dimensions, which are the means of the heights and widths of the two end sections.*

There is a number of alternative ways in which the prismoidal formula may be used, and some of these are given below, assuming in each case that the basic longitudinal spacing of the cross section is on chain, i.e. D is one chain (100ft).

(a) Treat each cross section as the end area of a prismoid one chain long, and estimate the dimensions of the mid-areas at the 50 link (50ft) points as the mean of the two corresponding dimensions in the end areas. This is difficult when the sections are irregular.

(b) Where estimation of the mid-area is difficult, arrange for extra section to be leveled at the mid-area positions as required. This means, of course, a large increase in the amount of field work.

(c) Treat alternate sections as end areas, i.e. the length of the prismoid is 2D. Unless the ground profile is regular both transversely and longitudinally, however, it is likely that errors will be introduced in assuming that a volume of earth is in fact prismoidal over such a length. Where this method is used, by taking $\mathbf{A_1}$ and $\mathbf{A_3}$, $\mathbf{A_3}$ and $\mathbf{A_5}$, $\mathbf{A_5}$ and $\mathbf{A_7}$, etc., as end areas of successive prismoids,

$$V_{1\text{-}3} = 2D/6\,(A_1 + 4A_2 + A_3)$$
$$V_{3\text{-}5} = 2D/6\,(A_3 + 4A_4 + A_5)$$
$$V_{5\text{-}7} = 2D/6\,(A_5 + 4A_6 + A_7)$$

Therefore

$$V = D/3\,(A1 + 4A_2 + 2A_3 + 4A_4 + 2A_5 + 4A_6 + A_7)$$
$$= D/3\,[\,(A_1 + A_7) + 2\,(A_3 + A_5) + 4\,(A_2 + A_4 + A_6)\,]$$

$$\mathbf{V = D/3\,[\,(A_1 + A_n) + 2\,(A_3 + A_5 + A_7 + \ldots + A_{n-2}) + 4\,(A_2 + A_4 + A_6 + \ldots + A_{n-1})\,]}$$

= D/3 [(sum of end x-section areas) + 2 (sum of remaining odd x-section areas) + 4 (sum of even x-section areas)] **…………….. (7.24)**

where **n** is an odd number.

This is **Simpson's Rule** for volumes.

(d) Calculate the volume between successive cross sections by the method of end areas, and apply correction to these volumes known as **prismoidal correction**. Such correction can be derived for regular section only.

If D = spacing of cross sections(A_1 and A_2)

h1 and h2 = the difference in level between ground level and formation at A_1 and A_2, respectively,

b = formation width

Prismoidal Correction For a Level Section is

$$P.C = D/6\, m(h_1 - h_2)^2 \qquad \text{…………… (7.25)}$$

Prismoidal Correction For Section with cross Fall is

$$P.C = \frac{D}{6}\, m\,(h_1 - h_2)^2 \frac{k^2}{(k^2 - m^2)} \qquad \text{……………(7. 26)}$$

Prismoidal Correction For Section part in Cut and Part in Fill is

$$\text{Fill P.C} = \frac{D}{12}\, k^2\,(h1 - h2)^2 \frac{1}{(k - m)} \qquad \text{……………. (7. 27)}$$

$$\text{Cut P.C} = \frac{D}{12}\, k^2\,(h1 - h2)^2 \frac{1}{(k - n)} \qquad \text{……………..(7. 28)}$$

NOTE : General Rule

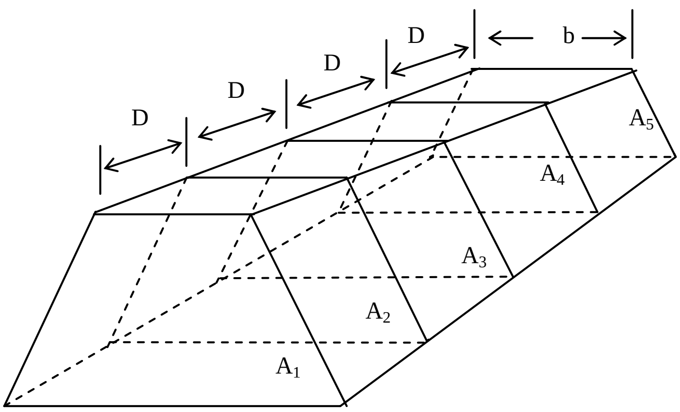
(1) *Calculate the cross-sectional area of the earthwork at regular intervals.*

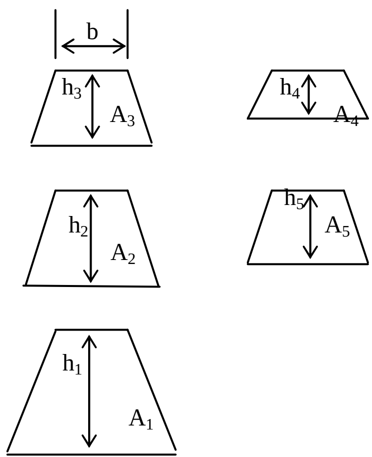
(2) *In cases where there is an odd number of sections, use those sections in Simpson's rule.*

(3) *In case where there is an even number of sections, use the maximum odd number of cross section in Simpson's rule and use the prismodial rule for the remaining prismoid.*

Example 7.11 : An embankment is formed on ground which is level traverse to the embankment but raising at 1 in 50 longitudinally so that five sections 50 ft apart have center line height of 20 ft, 18 ft, 16 ft, 14 ft, and 12 ft respectively above original ground level. If side slope of 1 in 3 are used, determine the volume of fill (yd^3) between the outer section when the formation width is 30 ft using end area method and Simpson's rule.

Solution :#





D = 50 ft
h_1 = 20 ft, h_2 = 18 ft, h_3 = 16 ft, h_4 = 14ft, h_5 = 12 ft
m = 3
b = 30ft

<u>For Section Level Across</u>

Cross-sectional Area = A = h (b + mh)

$\therefore$ $A_1 = h_1 (b + mh_1) = 20 (30 + 3x20) = 1800 \text{ ft}^2$

$A_2 = h_2 (b + mh_2) = 18 (30 + 3x18) = 1512 \text{ ft}^2$

$A_3 = h_3 (b + mh_3) = 16 (30 + 3x16) = 1248 \text{ ft}^2$

$A_4 = h_4 (b + mh_4) = 14 (30 + 3x14) = 1008 \text{ ft}^2$

$A_5 = h_5 (b + mh_5) = 12 (30 + 3x12) = 792 \text{ ft}^2$

<u>By End Area Method</u>

Volume of Fill = V = D [(average of end areas) + (sum of intermediate areas)]

$= D [(A_1 + A_5)/2 + (A_2 + A_3 + A_4)]$

$= 50 [(1800 + 792)/2 + (1512 + 1248 + 1008)]$

$= 50 [(2592/2) + (3768)] = 50 [(1296) + (3768)]$

$= 50 [5064] = 253200 \text{ ft}^3 = 253200/27 \text{ yd}^3 = \mathbf{9377.78 \text{ yd}^3}$

<u>By Simpson's Rule</u>

V = D/3 [(sum of end x-section areas) + 2 (sum of remaining odd x-section areas) + 4 (sum of even x-section areas)]

$= D/3 [(A_1 + A_n) + 2 (A_3 + A_5 + A_7 + \ldots + A_{n-2}) + 4 (A_2 + A_4 + A_6 + \ldots + A_{n-1})]$

$= D/3 [(A_1 + A_5) + 2 (A_3) + 4 (A_2 + A_4)$

$= 50/3 [(1800 + 792) + 2 (1248) + 4 (1512 + 1008)]$

$= 50/3 [(2592) + (2496) + (10080)] = 50/3 [15168]$

$= 252800 \text{ ft}^3 = 252800/27 = \mathbf{9362.96 \text{ yd}^3}$

7.5. Volume of Large Scale Earthwork

To determine large scale earthworks, fieldwork consists of covering the area by a network of squares and obtaining the reduced level. The volume is then determined either from the grid levels themselves or from the contours plotted therefrom.

7.5.1. Volume From Contour Lines

It is possible to calculate volumes using the horizontal areas contained by contour lines. Owing to the relatively high cost of accurately contouring large areas, the method is of limited use, but where accurate contours are available, as for instance, in reservoir sites, they may be conveniently used.

The contour interval will determine the distance D in the **' end area** or **prismoidal formula,** (Equation 7.20, 7.22, 7.23 and 7.24) and for accuracy this should be as small as possible, preferably 2 ft to 5 ft. The accuracy of the volume depends basically on the contour vertical interval. Generally, the closer the contour interval the more accurate is the volume. The area enclosed by individual contour lines are best taken off the plan by means of a planimeter.

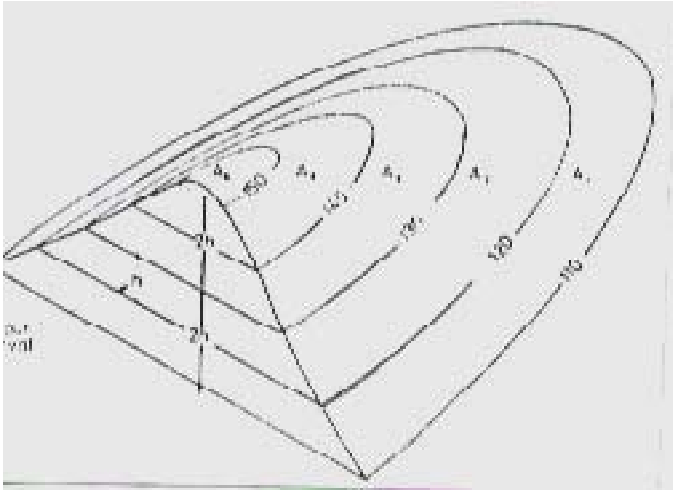


Fig 7. 23

In computing the volumes, the areas enclosed by two successive or regular successive contour lines are used in ‘**the end areas formula**.’

If required, **the prismoidal formula**, can be used, either by treating alternate areas as mid-areas or by interpolating intermediate contours between those established by direct leveling.

Example 7.12 : The areas within the underwater contour lines of a lake are as follows : -

Contour lines (ft)	290	285	280	275	270
Area (ft^2)	31500	24600	16300	8400	2050

Calculate the volume of water in the lake between the 270 ft and 290 ft contours (i) by end area method , (ii) by prismoidal formula.

Solution :#

contour interval is 5 ft, therefore D = 5 ft

(i) By end area method

Volume = V = D [(average of end areas) + (sum of intermediate areas)]

$$= D\left[\left(\frac{A_{290} + A_{270}}{2} \right) + A_{285} + A_{280} + A_{275} \right]$$

$$= 5 [(31500 + 2050)/ 2 + 24600 + 16300 + 8400]$$

$$= 5 [(33550/2) + 49300] = 5 [16775 + 49300] = 5 [66075]$$

$$= 330375 \ \mathbf{ft^3}$$

<u>(ii) By prismoidal formula (Simpson's Rule)</u>

Volume = V =D/3[(sum of end areas)+2(sum of remaining odd areas)+4(sum of even areas)]

$= D/3 [\ (A_{290} + A_{270}) + 2(A_{280}) + 4(A_{285} + A_{275})]$

$= 5/3 [(31500 + 2050) + 2(16300) + 4(24600 + 8400)]$

$= 5/3 [(33550) + 32600 + 4(33000)] = 5/3[33550 + 32600 + 132000]$

$= 5/3 [198150] = 330250$ **ft^3**

7.5.2. Volume From Sport Levels

This is a method by means of which the earthworks involved in the construction of large tanks, basement, borrow pits, etc., and similar works with vertical sides may be calculated. The computation is simplified if the formation is to be to a fixed level or to fixed falls, but even basements with several levels present little difficulty.

Having located the outline of the structure on the ground, the engineer divided up the area into squares or rectangles, marking the corner points as described in the method of contouring by spot levels. Level are taken at each of these corner points, and by subtracting from these the corresponding formation levels, a series of heights are obtained from which the mean heights of a series of vertical truncated prisms of earth can be found.

The volume of each prism is given by the plan area multiplied by the mean height of the prism. The prism may of course be considered as either rectangles or triangles as in shown in the following example.

Example 7.13 : Fig 7.24a Shows the reduced levels of a rectangular plot which is to be excavated to a uniform reduced level of 120 ft above datum. Assuming the sides to be vertical, calculate the volume of earth to be excavated different two methods.

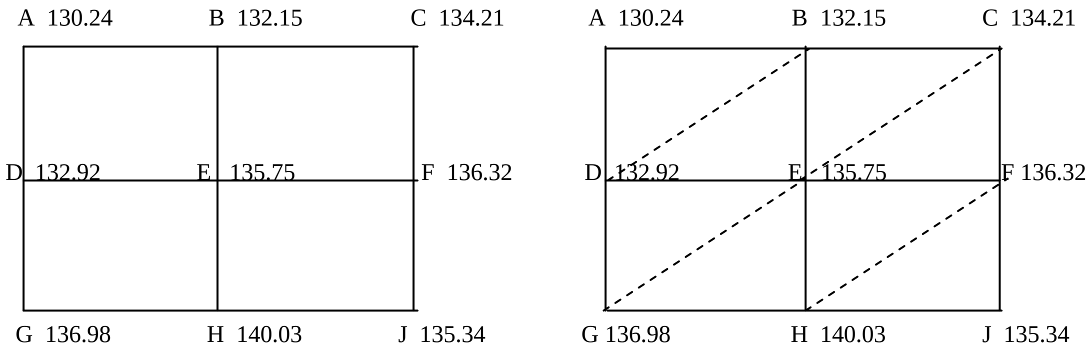
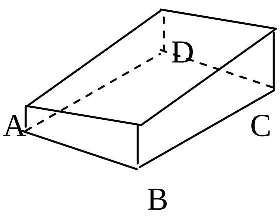


Fig 7. 24 (a) (b)

Solution :#

Draw the figure 7.24a
(a) Assume area is subdivided into four rectangle (Fig 7.24(a))

Station	Depth of Exc. (h_n)	Number of rectangles in which it occurs (n)	Product (h_n x n)
A	10.24	1	10.24
B	12.15	2	24.30
C	14.21	1	14.21
D	12.92	2	25.84
E	15.75	4	63.00
F	16.32	2	32.64
G	16.98	1	16.98
H	20.03	2	40.06
J	15.34	1	15.34



h_n x n = 242.61
total mean height of prism = 242.61/4 = 60.6525

Volume = V = (plan area) x (total mean height of prism)
= (length x width) x (total mean height of prism)
= (50 x 40) x (60.6525) = 121305 **ft^3** **=** 121305/27 = 4492.77 **yd^3**

Draw the figure 7.24b

(b) Assume the area is divided into triangles as shown by the dotted lines in Fig 7.24(b)

Station	Depth of Exc. (h_n)	Number of triangle in which occurs (n)	Product h_n x n
A	10.24	1	10.24
B	12.15	3	36.45
C	14.21	2	28.42
D	12.92	3	38.76
E	15.75	6	94.50
F	16.32	3	48.96
G	16.98	2	33.96
H	20.03	3	60.09
J	15.34	1	15.34

h_n x n = 366.72
total mean height of prism = 366.72/3 = 122.24

Volume = V = (plan area) x (total mean height of prism)
= (½ (base x height))x (total mean height of prism)
= (½ (50 x 40)) x (122.24)) = (1000) (122.24)
= 122,240 ft3 = 122,240/27 **yd3** = 4527.41 **yd3**

PART - III

CHAPTER - 1
UNDERGROUND TRAVERSING

1.1 General

Running an underground traverses differs essentially from running a surface traverses in both the method of setting up the transit and the information to be obtained. Because of restricted illumination, greater care must be exercised in reading the instrument and tape. Moreover, in the immediate vicinity of the station, the walls and back must be carefully inspected for loose rock; unexpected falls of rock may injure the transit or its operator, or an important station may be disturbed or made inaccessible.

In setting up the transit, avoid placing the legs on rails, on ties, on loose material buried in the drift, or in the drainage ditch. Because of the lack of light these mistakes are easily made. A common mistake is getting a tripod leg against a rail; disturbance of the rail may move the transit from under the station. Many times the transitman does not realize this has happened. Yet on short shots, a sight displacement would cause considerable angular error.

1.2.Station Marker and Spads

Underground stations are usually placed in the back or roof of the heading. To accomplish this, a hole is drill, either with a machine (stopper, jackleg, or jackhammer) or by hand, a wooden plug is inserted, and into this the spad is driven. Quite often part of a drill hole remaining after blasting may be used. The station plug hole does not have to be vertical.

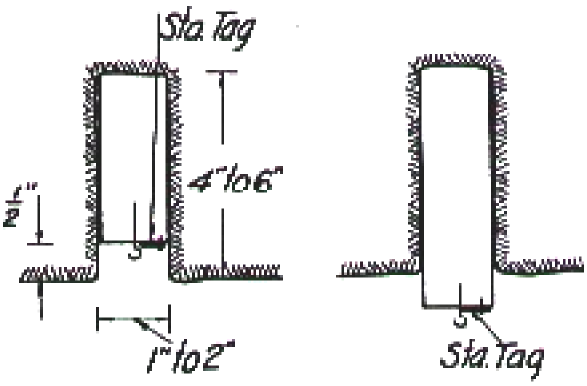


Fig 1.1 Method of putting in station plugs

Figure 1.1 shows the method of putting in a station. The plug must fit tightly. The method indicated by Fig 1.1a should be used whenever there is danger of the station's being hit by flying rock disturbed by workmen, some of whom have the bed habit of fastening a pipe line of a power or light line to a survey station if the plug projects far enough.

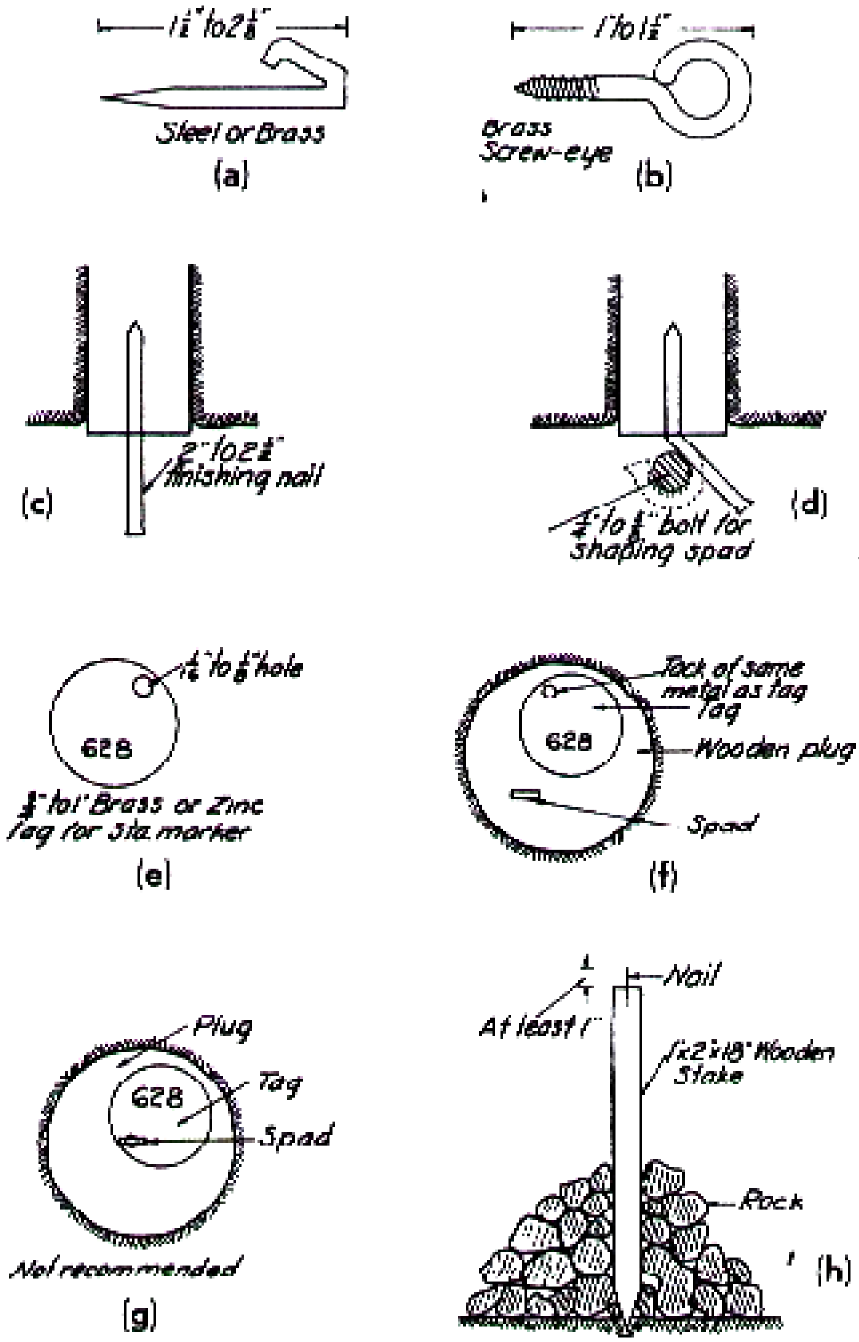


Fig 1.2 Spad and station markers

Many type of spads are used. Those shown in Fig 1.2, a and b, are satisfactory. That shown in 1.1a can be obtained from nearly all instrument and equipment manufacturer; and the brass screw eye can, of course, be bought at any hardware store. The spad should be aligned with narrowest dimension in the plane of the line of sight. Fig 1.2, c and d, illustrates how a spad may be formed from a finishing nail. Fig 1.2, e, f, and g, showed a common type of station marker. Because of electrolytic action the tag should not be fastened to the plug by the spad. Under damp or wet conditions one of the two (dissimilar metals, which they invariably will be) will deteriorate. It is preferable to have the tag and the spad separated as far as space permits. Fig 1.2h shows a method of putting in a temporary station when the back is too high to reach, or where a permanent station is not necessary (stope surveys, drift detail, etc.). Several of these stakes are made up and carried at all times in the equipment sack.

Here are a few words of caution on selection and placing of underground stations: (1) The back should be carefully inspected for loose rock. A place may appear entirely suitable before a hole is drilled for the plug and yet the vibration of the drill may alter this condition so that at a later date the instrument or its operator may be seriously injured. (2) Whereas it is quite often necessary to place the spad in timber, such a station is nearly always unreliable. The blocking may become loose; allowing the set to move, or heavy ground may displace the set. When accurate control is required, the position of such stations must be checked at frequent intervals. (3) When a station is placed in the floor, the back should be inspected to guard against a fall of loose material before the setup is vacated.

1.3. Numbering Stations

Assigning numbers to underground survey stations is source of irritation to most engineers. If the mine development is relatively simple (a continuous vein, mineable from one drift following the strike with no gaps along the dip would be an example), little difficulty is experienced. On the other hand, a mine requiring numerous parallel drifts, crosscuts, inclines, raises, etc., for searching out and developing the ore, and lacking continuity along both the strike and the dip, presents an entirely different problem. Where only the higher grade ore is extracted, with later backing up and removal of k\low grade ore, the operations throw the numbering system into confusion, because continuity of numbers cannot be maintained. Ordinarily the practice is followed of assigning a serial number to each level in order to provide differentiation between levels. For example, the 100-ft. level stationing will be 101, 102, 103, etc., and the 200-ft. level 201, 202, 203, etc. That is, each 100-ft. level is designated. If several levels are skipped say, the first level started at 400 ft. below the collar, it would still be the 400 ft. level, and the stations would be 401, 402, 403, etc. Extensively developed mines may soon pass beyond the 99 station numbers available. When this happens, a hyphenated system may have to be applied, using the level designation to fix a station’s location. Mines operating through adits or tunnels may be handled by assigning the equivalent level number to the tunnel or if only one tunnel is used by using stations with number from 1 to 99.

The level stationing should be applied with number to the main traverse. Crosscuts and secondary drifts are usually given numbers or names, which definitely fix their horizon. With their designation as a prefix, stations are started at 1 and continued numerically. They usually extend to only a few stations. For example, crosscut 9 is being driven. The first station in the crosscut would be presented by XC9-1. Or, if crosscuts are driven on both sides of the main drift, which has a more or less easterly direction, the north-side crosscut stationing would be NXC4-12, and the south SXC2-6.

Station for crosscuts off of crosscuts becomes very involved. Foe example, the north crosscut designated above has a drift to the east. Here the station might be recorded as N-E-XC4-2.

Some systems simply start out with 100 and increase as each station is run in, regardless of level or location. Beyond any question this simplifies the numbering. After a while, however, it is impossible to tell which level station is on without the aid of the map. Also stations are apt to be duplicated or skipped, which may cause serious mistakes. This method should be avoided whenever possible.

The most satisfactory way of designating raises and winze is by their block number. Stopes and the survey stations in them may be coordinated with the level from which the stope is first entered. For example, the first stope starting from the 500-ft level would be the 501 stope, the second one the 502, etc. The stationing in the stope would be 501-4, etc. Or the block number for starting place of the stope might be used.

Level intermediate between main levels are usually attached, for identification purposes, to the level from which the raise or winze starts. For example, a raise is driven from the 700-ft level. At a certain height drifting is started. This drift would be called the 700-ft intermediate. If it were the fourth, such drift to be undertaken. “No.4-700 ft intermediate” would locate it. The first station in this drift would be No. 4-700-ft-1.

Sometime the numbering of the intermediate drift is made to depend upon the level to which it is closer.

Whatever system is finally decided upon should be one which is easily understood by new men and one which permits, so far as possible, the consecutive numbering of stations as the work progresses. Side surveys off the main heading are better assigned a separate series of numbers rather than variation of the main traversing numbering.

The foregoing is offered not as the last word in numbering underground survey stations but merely as a guide to the engineer first confronted with this vexing detail of the mine surveyor’s duties.

1.4. Setting up the Theodolite

Underground settings differ from surface settings in that practically all are under the point instead of over it. Only in wide headings such as railroad tunnels would it be practicable to place stations in the floor, and even there such practice is not desirable. Because of experience the operation at first appears slow and awkward; yet after a little practice it can be made in less time than over a point. The optical type instrument may be equipped with optical plumbing for viewing an overhead station.

Before ever attempting to set up a theodolite, whether underground or on the surface, the height of the leveling screws should be equalized and the head motion centered. Absolute adherence to these suggestions will save much wasted time. It is a good ideal also to align one pair of leveling screws along the backsight.

The plumb bob should be hung on the spad by means of the plumb bob. It should be height enough at the start so as not to bump the theodolite. The theodolite is placed under the bob and the legs are firmly pushed down. Previous to this the vertical circle should be set at zero. This is done without the reading glass. On top of the telescope is a centering mark directly over the center of the vertical axis. This must be centered under the plumb bob Figure 1.3 shows the instrument set up. Too much time should not be used in trying to get the center mark located correctly through the medium of picking the instrument up and moving it around; within three or four inches is close enough at first. Next unclamp the proper leg and raise or lower the upper section, bringing the mark under the plumb bob. Now roughly level up by means of the legs recenter. A little experience will indicate the position for the center mark; usually it must pass a little beyond the point of the plumb bob, since the leveling operation will bring it back. After a sufficient number of tries the final adjustment will fall within the range of the head motion.

The question naturally arises as to how closely the plumb bob point should be centered over the center mark. This depends on the length of the shot (backsight and foresight) and the precision desired. For example, with a 100-ft shot, the instrument could be off the point 0.029 ft before causing an error of one minute. Yet at 20 ft, about 0.006 ft will cause this error. The safest thing is to get the plumb bob point within the punch mark. As the instrument would usually be off for both backsight and foresight, the error would be greater than indicated above, where only one pointing was assumed.

Figure 1.3 indicates the measurements that are made to obtain the HI and occasionally the down (D).

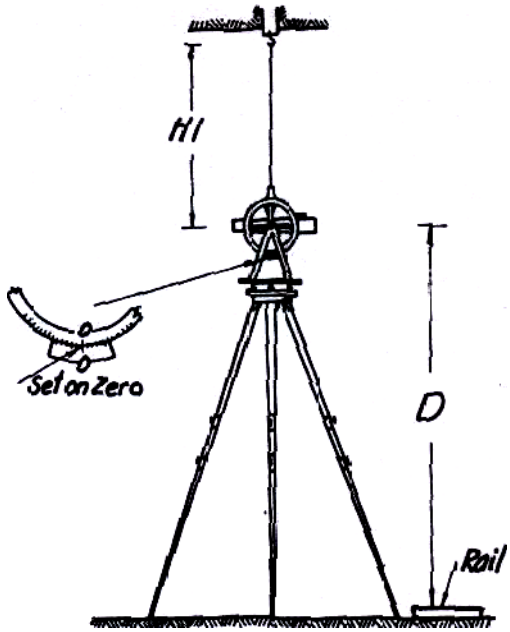


Fig 1.3 Setting up the theodolite

1.5. Selecting station location

Stations will usually be located as permanent points. In some cases the nature of the ground will make many of them semi permanent. That is, frequent checking for position will be necessary because of ground movement. In some mines, stations will only occasionally have to be placed in stulls or caps or possibly other forms of timbering. Many mines are entire timbered. There are several reasons for timbering mine working and these have a bearing on the stability of survey station. In those mines where timbering is used merely to prevent an occasional fall of loose rock, caused by air-slacking or vibration from blasting, the station may never require checking, provided the set is tightly blocked before placing the station. Other mines require timber because of heavy, moving ground. Here the timber shift around and the station with them. Permanent base lines should accordingly be established, when possible, in ground where movement does not occur. This can usually be done so as to avoid plumbing the shaft too frequently.

Figure 1.4 shows the proper location for a station to serve more than one drift. A layout of this is of common occurrence. Full advantage of the possibilities must be taken to avoid setting too many stations for entering the drifts. The “a” stations are poor the new foresight, 1, 2, 3, cannot be seen from a single setup.

The instrument must not put too much dependence on his helper when it comes to selecting locations. In many mines the helper is taken from the shovelers or trammers when need arises. The same man may always be obtained, but nevertheless such a helper is seldom really effective, since he cannot be expected to choose good setups. The possibility of error is always present because of poor setups, and too many of them arise without carelessly making more.

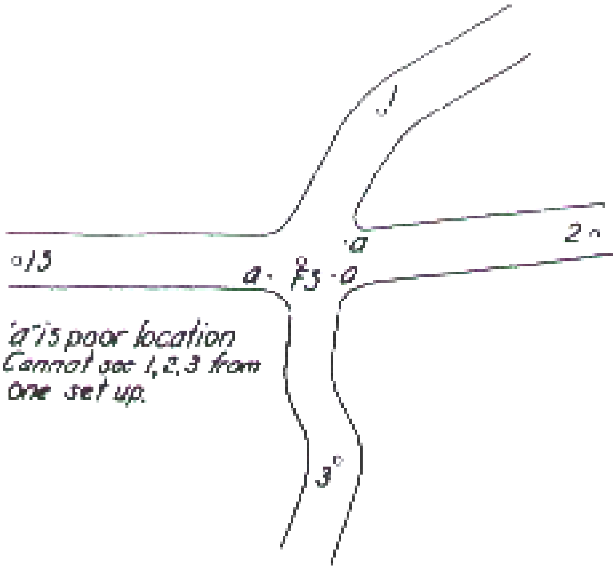


Fig 1.4 Choice of foresight station

1.6. Angle and measurement

The novice is usually more or less confused with his first few attempts to read angles by artificial illumination. A reflector back of the vernier cover glass is almost indispensable. Most mine theodolites have this attachment. Whether of celluloid or of glass it sooner or later becomes destroyed. If this happen, apiece of glazed paper inserted in the support is of considerable help. It will be difficult to read the vernier without it. The light from the cap lamp or other illuminant should be directed between the cover glass and the reading glass. The best position is a matter of trial and error. After a little experience the operator finds the correct location. For this sort of work the handheld lamp is most satisfactory; with a slight hook bent on the bail of the carbide lamp it can be hung from the shoulder. Both the operator's hands are free, and by stooping he can bring the light to the desired level.

When setting stations, one should double the angle. The second reading is taken with the telescope plunged. This compensates for misadjustments of the standards, for collimation, and for index error. Angles should double to a plus or minus one minute. If one does not, the operator should repeat the observation and satisfy himself that the fault lies not with this manipulation but with the instrument. The average of the double angle is the correct one. With the theodolite in fair adjustment and vertical angles less than 5º, no difficulty should be experienced in getting the check. For the usual mine traverse, a recording to the nearest minute is sufficient.

Probably most mine survey crews for routine work consist of only two men. For special work, such as shaft plumbing, extra help is picked up from the mine crew. It is imperative that the instrument man organize his work so that he makes all of the important readings. It is dangerous to delegate this to the helper. He should read and record in the notebook the HI, the HS (height of shot; sometime called HP, height of point), and the SD. The zero end of the tape is taken by the assistant. In this way the engineer makes the reading and depends on the assistant only for holding on zero. If the check measurement varies by more than 0.01 ft, the tape probably was not held on zero. A satisfactory point to observe and measure to is usually where the string enter the plumb bob, though in steep shots the point of the plumb bob may have to be used. The helper must be cautioned against disturbing the foresight plumb bob after measurement have started. When the instrument is moved to this station, the HS is read.

Note: It is bad practice to unreel the tape along the drift in anticipation of taking the slope distance. Member of the survey party, or other workmen, are very apt to step on it. Moreover, in wet drifts to wind all of the tape up, and the scouring action, with the effect of rusting, soon makes the graduations extremely hard to read. A 200-ft steel tape can be stretched practically its full length and still be kept off the floor of the drift. If this cannot be done, the tape should be wiped off each time as it is wound up. Dragging the tape underground as in surface surveys is out of the question.

The most satisfactory procedure for running a traverse is as follows:

1. Set up the theodolite.
2. Record HI.
3. Record right and left.
4. Set on zero and take backsight with lower motion.
5. Unclamp upper motion and set on foresight. In windy drifts the swing of the plumb bob may have to be bisected. It is usually possible to reduce the amplitude of the swing to about 0.01 ft by holding the tracing cloth under the bob with the point just making contact. As the horizontal angle is of greatest importance, the horizontal cross hair should be set at the top of the plumb bob first. Then adjust the vertical hair for the horizontal angle.
6. Read and record HA first; unclamp lower motion and turn vertical circle to face the operator and read VA (if conventional transit is used).

7. Plunge telescope and set on backsight, using lower motion.
8. Release upper motion and sight foresight.
9. Read HA and VA. On flat angles, reading the VA the second time is not necessary. If HA fail to double, repeat process, after setting on zero and placing telescope in direct position.
10. After all of the angular measurements are completed, the helper takes the zero end of the tape to the foresight station and the SD is measured. After the first reading the tape is slacked off and a check reading is taken. Before making the measurement the theodolite must be pointed in the direction of the foresight station.
11. Move to the foresight station and record HS (and if necessary the down) before setting up and disturbing the position of the plumb bob.

All of these operations can be performed while holding the notebook under the left arm with the pencil marking the place. A considerable amount of time is thus saved. Too many surveyors waste time looking in each pocket for the notebook, then the pencil, and finally finding them on the ground, in a crevice in the rock, or on a timber. Another convenient way is to insert the book in one of the split legs. Many times work must be done between the passing of trains or trammers, or a mechineman must stop drilling until the station is located; this usually means that each second will have to be utilized to best advantage.

1.7. Taking detail

By detail is meant all the irregularities and bulges along the line of the surveys. The detail in stopes will be studied later. The correct recording of irregularities in drifts, crosscuts, raises, winzes, etc., is essential for several reasons. Tracings of most mine maps are made at some time or other, and are used underground, where the geology is recorded on them. Also the mine management may wish to drive raises from one level to another, and lack of knowledge of slight curves in the drift will cause the raise to come up in the center of te drift or too far to one side. The geologist’s task is very harder if the actual underground workings are different from those indicated on the map faults, slips, contacts, veins, etc., do not plot right. He has to remark the map; his work is materially slowed down and its value is desired.

Irregularities arise from the map nature of the rock being penetrated, from the experience of the drill runner, and from his following indications of ore that look promising. Unless the miner is expert, blocky ground, like some limestones and igneous rocks will cause a sinuous opening of varying width. Whatever the cause, departures from the required direction and size must be recorded. Such measurements are taken to the nearest one-half foot; in exceptional cases when the plotting is on a very large scale, more precise measurements are taken Rights and lefts are taken facing the FS station.

There are two methods of taking detail: the angle right method and the offset method.

1.7.1 Angle Right method

There are several things in its flavor. In the first place, as pointed out before, the survey party will usually consist of two men. Now it is difficult to take offsets with a two-man party. The drift outline is more accurately taken, and the data are readily plotted on the map.(see Fig 1.5 and 1.6).

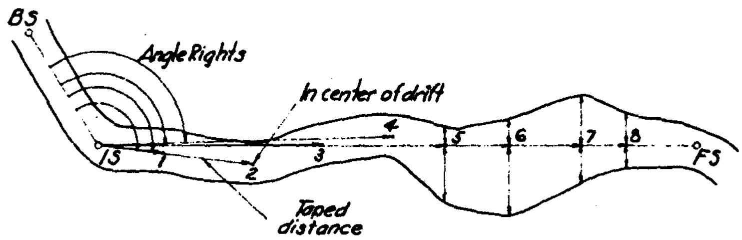


Fig 1.5 Detail by angle right

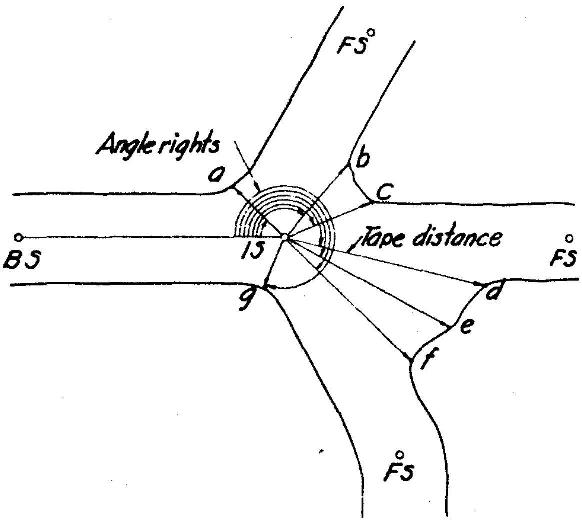


Fig 1.6 Detail at the intersection of drifts

In Fig 1.5, after setting the FS, the operator sets the plates on zero and takes a new backsight. The helper then starts at the first curve in the drift beyond the instrument station; he holds his light in the center of the drift, the engineer locating it with an angle right to he nearest 15' (if the data can be plotted closer than this, the readings should be accordingly refined). After the angle is turned, a mark is made on the wall with the carbide lamp or a carpenter's crayon (keel). The helper then move to shot 2. This operation is continued until the foresight station reached. Horizontal distances from the instrument station are now taped to points in the center of the drift opposite the marks. If the width of the drift at these points differs materially from what it should be, rights and lefts are taken. In figure 1.1.5 points 4, 5, and 6require right and left. On the map the data are plotted with a protractor and scale.

Another common type of drift detail is shown in figure 1.6. This arises where several drifts join. After setting the FS, set the plate on zero. Each FS should involve a separate setting on zero; trying to double a series of consecutive angle rights under these conditions will lead to complications, since each preceding angle must be subtracted; the compensation gained by plunging the telescope is not properly distributed, and it is lost altogether if the angle is backed up each time to get the double. Points a, b, c, d, etc., are measured, and the distance is taped. For this sort of detail it is not necessary to sight the lamp through the telescope just point the telescope and sight over the top nor is there any reason for clamping the motion.

Detail by the angle method is taken easily and rapidly. It is just as conveniently plotted.

1.7.2 Offset method

Very little need be said this, since it is common practice in plane surveying. Figure 1.7 gives the method. The main difficulty arises if there is not a sufficient crew. With three men it would in the illustration given, be superior to the angle method. The conditions in Fig 1.6 are better met, however, by the angle method. Fig 1.7 is plotted on the map by laying the scale along the BS-FS line and marking off the distances; the offsets are then plotted at right angles to this line.

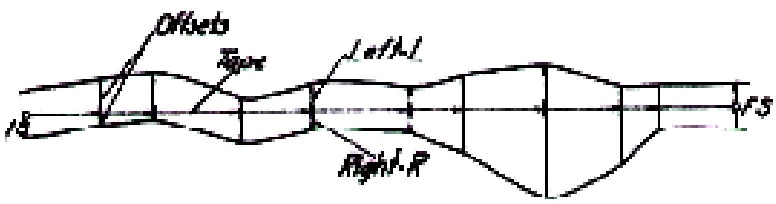


Fig 1.7 Detail by offsets

Deflection angles could be used. The possibility of error in recording right or left angles is against their application.

1.8. Elevation

There are three ways in which elevations can carried through underground workings; by theodolite and tape; by differential leveling; and by taping down a vertical shaft (a special case, only rarely done).

1.8.1 Theodolite and tape

This method is by far the commonest. With ordinary care in measuring the HI, the HS, and the vertical angles, elevations suitable for most underground control can be carried a considerable distance without accumulating too great an error. The field work takes very little more time than running a traverse for direction and distance. The office work is about one-third greater. For connections requiring traverses of more than a few hundred feet, differential leveling should be employed.

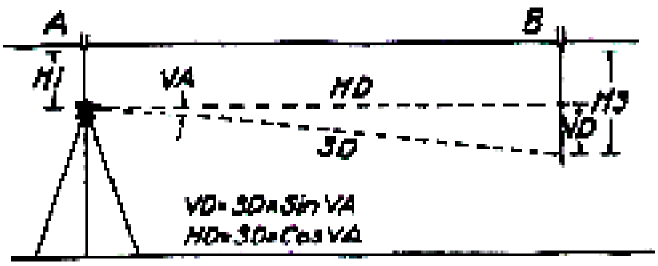


Fig 1.8 Principle of taking elevations with theodolite and tape

Fig 1.8 represents diagrammatically the theodolite and tape method of carrying elevations. The fundamental formula is :

Elevation of FS = Elevation of IS ± HI ± VD ± HS ………… (1.1)

For nearly all underground stations this can be written as follow (Fig 1.9)

Elevation B = Elevation A - HI ± SD sin VA + HS …………… (1.2)

= Elevation A - HI ± VD + HS ..………….. (1.3)

At all stations where the theodolite is under the point, the HI is subtracted because the instrument is lower than the station. For a setup over the point, the HI is added. The sign to accompany the HS is occasionally confusing to the beginner. But it is necessary to remember only that, whenever a plumb bob is sighted, the HS must be plus, for the very simple reason that the station must be above the plumb bob. When a stake is used for a foresight, the HS is usually considered zero (an exception is in stope surveys, where the HS indicates the elevation of the ore at that point). The elevation of the floor of the drift is of no particular value, except for occasional profiles.

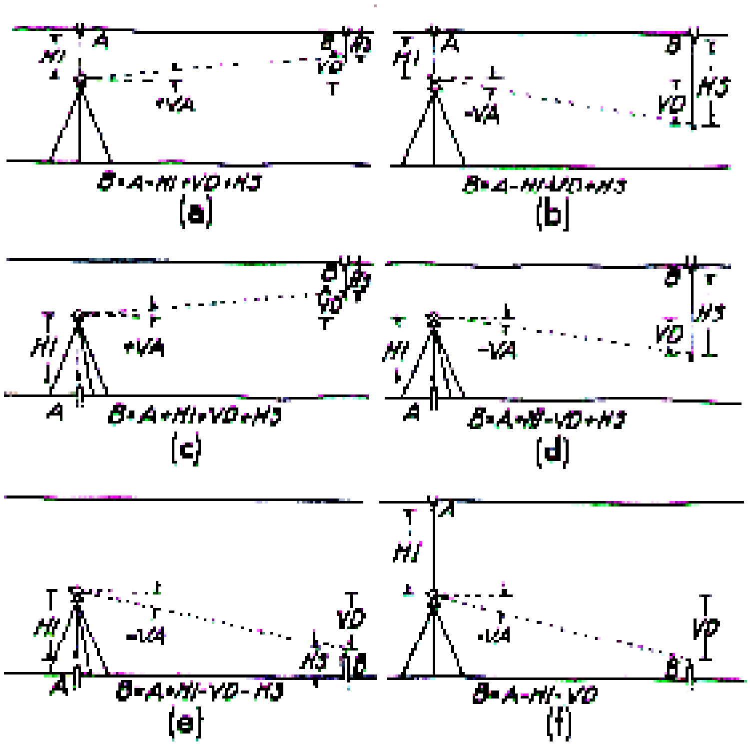


Fig 1.9 Elevations with theodolite and tape

If the vertical angle is plus, the vertical distance (VD = SD sin VA) is plus. The plus angles means that the point to which the angle is measured is higher than the instrument. We now see why emphasis has been placed on reading the vertical angles as plus or minus: if the wrong sign is inadvertently recorded, the elevation is off an amount including twice the vertical distance. In a stope survey this nay amount to a considerable tonnage of ore. Figure 1.9a to f, shows conditions commonly encountered.

The HS is measured from the bottom of the sped to the top of the plumb bob (if this is the point to which the VA is turned); and the HI from the bottom of the spad to the mark at the end of the telescope's horizontal axis. When a stake is employed, the top of the nail is ordinarily used.

1.8.2. Differential leveling

Underground leveling with rod and spirit level is very similar to surface work. There are a few precautions to be observed. At first it may be somewhat difficult, as the rodman must move his light up and down the rod until it is opposite the required reading. If he uses a carbide lamp, he must be careful not to blister the markings on the rod. The use of a target is totally unnecessary, as the levelman can estimate to the nearest 0.002 ft. The rod is placed upside down against the spad: in doing so the rodman must be careful not to disturb the spad's position. When carrying elevations on stations placed in the back, the backsight and foresight have signs just opposite those of similar work for surface surveys the backsight is minus and the foresight is plus. Otherwise the same types of notes and calculations are as followed as for surface leveling. Underground levels can be carried with a closured of $0.05\sqrt{miles}$ as readily as on the surface, and this standard of precision should be set.

Most mine surveyors use differential leveling only for precise work involving long and difficult connections. Some mine engineers follow the practice of carrying elevations with the level at definite time intervals, this frequency depending on the amount of underground traversing and the use to which the elevations are to be put.

1.8.3. Taping down the vertical shaft

When entrance is gained to the mine by a vertical shaft, the elevation can be transferred to levels below the color by measuring down the shaft. This is usually done by taping or by measuring the amount of hoisting rope unwound from the hoist drum. The latter is very uncertain and cumbersome, and should be used only for rough values.

The surface elevation is carried to a permanent monument close to the color of the shaft. From here it is transferred to a heavy spike placed in the color set or to one of the rail ending at the color. If the level interval is less than the length of the tape available, the vertical distance from the color rail to the rail or other point on the level below is measured directly. Subtracting the tape distance from the surface elevation gives the level elevation. This value should at once be transferred to a permanent survey station. If the length of the tape is less than the level interval, the cage must be used. The bonnet is lifted and the engineer gets on the cage with the tape. Then the cage is slowly lowered while the helper holds the zero end at the upper station. When the end of the tape is reached, the cage is stopped and a nail is driven into the timber. The nail is now raised to the surface, the engineer at the same time winding up the tape. Both men now are lowered to the intermediate station. The assistant remains there on the timber, placing planking across the sets if necessary, and the engineer is lowered to the level. The distance between levels is not apt to be off more than a few hundredths of a foot an error of no importance. If exceptionally exact work is required, corrections must be made for tension and temperature of the tape and for verticality.

CHAPTER - 2
TRANSFERRING THE MERIDIAN: TUNNELS AND INCLINES

2.1. General

Transferring the true meridian from the surface control to an underground base is one of the most important and often one of the most difficult and exacting operations the mine engineer has to perform. He will be called upon not only to take the meridian down a shaft but also to carry it up raises and underground shafts to workings at a higher elevation than his first base. An accurate transference of the azimuth is necessary in order to orient the underground work with the surface workings and boundaries and sometimes more important to know the relative position of various levels.

The more extensive the workings from the original base, the greater the care that must be taken. If the mine operations are restricted to but a few thousand feet along the strike, it is evident that the same precision need not apply as for workings running for several miles.

For the alignment of long railroad tunnels, waterworks tunnels (some of which are well over 30 miles in length), highway tunnels, and subways under rivers, where metal linings must accurately meet, it is at once apparent that a different technique must be applied. The basic principles are the same as for ordinary mine work. Generally a 10 or 20 sec. transit or optical-reading theodolite will be used instead of the mine surveyor's 1 min. or 30 sec. instrument. More time is taken for the operations; a larger crew is made available; and several parties check against each other. Except in a few of the largest and most extensive mines, these ideal conditions are impossible. Fortunately, most mining work does not require such precision.

The procedure for transferring the meridian depends on the type of mine entrance: (1) tunnel (adit), and (2) inclined opening.

2.2. Tunnel Method

This method requires very little discussion other than that already given under traversing. Many tunnels are quite long; therefore, it may be necessary to repeat the angles more than twice; linear measurements may have to be corrected for tension and temperature and methods simulating base line measurements followed. In most instances such refinements are not made, but greater care is exercised than in ordinary traversing. It is advisable to carry the elevation by differential leveling. Shots in excess of 200 ft. can usually be taken. The limit will be the distance at which the plumb bob or specially illuminated target can be observed with the illuminating facilities available. The taping may be done in several ways. One is to project the instrument station and the foresight station to the floor and tape between them. This is not especially recommended, since it is difficult to correct for inclination and the measurement will be affected by uncertain temperature variations in addition to the uncertainty of the projected point when the tape is stretched on the ground. A series of plumb bobs (by using stulls across the tunnel) at intervals slightly less than the length of the tape may be set on line and adjusted to the same horizontal position. Unless the tunnel is exceptionally windy, this is satisfactory. Another way is to tape between stakes or tripods in pretty much the same manner as in making base line measurements for triangulation. In making the angular measurements, plunging of the telescope must not be forgotten.

There is no reason why a precision of 1 in 10,000 should not be attained in these long tunnel surveys without resorting to tension and temperature corrections. A 1 min. transit is sufficient, although if angles are repeated more than twice, time would be saved by using a 30 sec. instrument. Running a return check survey should be considered and the discrepancy between the two distributed.

2.3. Inclined Openings

Many mining operations are carried on through inclined workings. When the inclination exceeds about 50°, the transit with the customary fixed telescope is no longer convenient to use. (The angle depends on the transit. Some instruments with low standards and large diameter plates have even less range than 50°. Others may get up to 60° or more. It quite often happens that a steeper plus vertical angle can be read than minus.) For surveying in steeply inclined workings, the engineer has available the auxiliary telescope.

The auxiliary telescope may be used in two ways: as a side telescope and as a top telescope. A modern transit has the auxiliary interchangeable, since conditions occasionally call for the use of each. If the instrument is to be used primarily for controlling the driving of inclined openings, where the inclination is of secondary importance but the azimuth must be carefully controlled, the top telescope will prove most desirable. If azimuth control matters little but inclination must be carefully watched, then the side telescope is preferable. If the purpose is principally transferring the meridian which is not a daily routine the side telescope is better. By all means get the side telescope if any doubt exists. As a matter of fact, the author can see but little reason forever using the top telescope for other than the daily routine of controlling horizontal angles. The top telescope must be in absolutely perfect alignment with the main telescope or else complicated adjustments must be made, whereas in the side telescope the lack of alignment in both the vertical and horizontal planes can be compensated for by instrumental manipulation. Methods for carrying the azimuth through inclined openings when an auxiliary telescope is not available are discussed on Article 2.7.

2.4. Side Telescope

It will be noted that a compensating weight is necessary on the opposite end of the telescope axis. So far as the author knows, auxiliary telescopes are attached to the instrument with but one adjustment possible. (Some early types permitted only a cross hair adjustment.) This adjustment permits the alignment of the horizontal cross hairs in the side telescope and of the vertical hairs in the top telescope. Parallelism of the vertical hair is supposed to be fixed in the side telescope and of the horizontal hair in the top telescope by the manufacturer. Continual use causes a slight wear in the attachment, so that after a while parallelism can be obtained only by shifting the respective cross hairs. The attachment should permit adjustment in both planes.

In setting up the transit with the side telescope attached (and this applies equally to the top telescope), take care that a leg of the tripod is not in the line of sight. As the side telescope is used in both the direct and the inverted position, the positions of legs on each side of an instrument equipped with it must be inspected. Since the top telescope cannot be used in the plunged position, one leg is all that might interfere with the full use of an instrument so equipped. Finally, a setup close to a wall quite often obstructs the view when the side telescope is plunged; this must be guarded against.

There are two methods of using the side telescope: in adjustment with the main telescope, and out of adjustment with it.

2.4.1. In Adjustment

This method depends on the accurate alignment of the side telescope with the main telescope. Before attaching the auxiliary, check the adjustment of the transit. Vertical collimation, standards, plate levels, and index error must be ascertained. Check the collimation over a distance somewhat greater than the length of the underground shot. Adjust the standards at the steepest possible vertical angle, using the main telescope. After making sure that the instrument is in adjustment, attach the side telescope and counterbalance. Align the side telescope on a target whose distance is approximately equal to that to be observed in the mine. First, set the horizontal hair on a definite point. Then turn the transit until the side telescope is on the mark. Raise or lower the side telescope, by means of the adjustment provided, until the horizontal hair coincides with the mark. The horizontal hairs are now in the same plane.

The next step is to determine the eccentricity of the telescopes, or the distance between the lines of sight. This is indicated by *x* in Fig. 2.1. At a distance *A* a scale is mounted in a horizontal position. The intercept *D* between the two vertical hairs is read. At *B* distance beyond *A* the intercept *C* is similarly found. If *C* is greater than *D*, the lines of sight are diverging (Fig 2.1a), while if less they are converging (Fig 2.1b). The appropriate calculations are as follows :

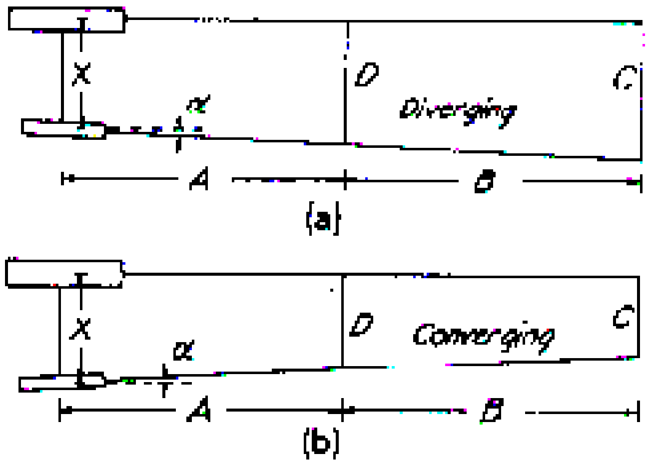


FIG 2.1 Determination of eccentricity.

Diverging lines of sight: *x is* easily found by simple proportion.

$$A:B :: (D\text{-}x):(C\text{-}D)$$

$$\frac{A}{B} = \frac{(D-x)}{(C-D)}$$

$$x = \frac{AD+BD-AC}{B}$$

$$\alpha = \text{angle of divergence} = \tan^{-1} = \frac{(C-x)}{(A+B)}$$

Converging lines of sight: here the proportion is

A : B :: (x- D) : (D – C)

$$\frac{A}{B} = \frac{(x-D)}{(D-C)}$$

$$\alpha = \tan^{-1} = \frac{(x-C)}{(A+B)}$$

If there is no appreciable difference between D and C, the lines of sight are parallel. It should now be evident why (A + B) should closely equal the slope distance to be measured underground. In case D does not equal C and the attachment has no means of adjusting the telescope, parallelism must be achieved by shifting the vertical cross hairs. On a much used and badly worn instrument it may be impossible to move the cross hairs far enough, and in that case the adjusted method cannot be used. Of course after making the cross hair adjustment, x must be redetermined.

Assuming, for the time being, that the telescope can be lined up, the procedure for the underground work follows. First, the example illustrated by Fig. 2.2 will be solved. Substituting in the formula for *x*,

$$x = \frac{AD + BD - AC}{B}$$

$$= (100 \times 0.42 + 100 \times 0.42 - 100 \times 0.51) / 100$$

$$= 0.33 \text{ ft}$$

When *A* = *B*, as above,

x = 2D – C, for diverging hairs,

= 2 x 0.42 - 0.51 = 0.33 ft. Or, *x* may be found as shown in Fig. 2.2

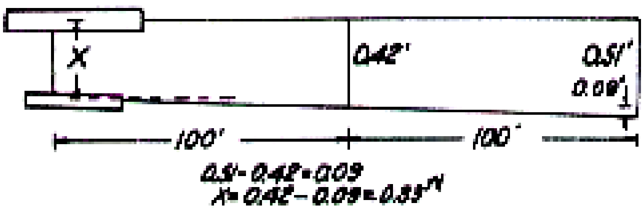


Fig 2.2. Calculation of eccentricity.

The principle involved in using the side telescope in adjustment is the solution of a rectangle whose end dimensions are *x* and whose sides are equal to the horizontal distance (HD = SD cos VA). What we wish to find here is the angle that a diagonal in the rectangle makes with the side. This is θ in Fig. 2.3a. The full lines represent the telescope in the direct position. In this position it is on the right-hand side. By the dotted lines the plunged position is shown. In each case x is the same, provided nothing about the transit has been changed.

A few remarks follow on transporting the instrument underground. Ordinarily a transit with the side telescope mounted is cumbersome and awkward to handle. Unless the scene of operations is easily accessible, it is safer to dismount the telescope and use the carrying case. When it is reassembled, recheck the alignment. Why waste time, then, with the first check on the surface, if it is now to be repeated? Because the recheck, in dark and cramped quarters, is minor, whereas a major check needs optimum conditions for accuracy.

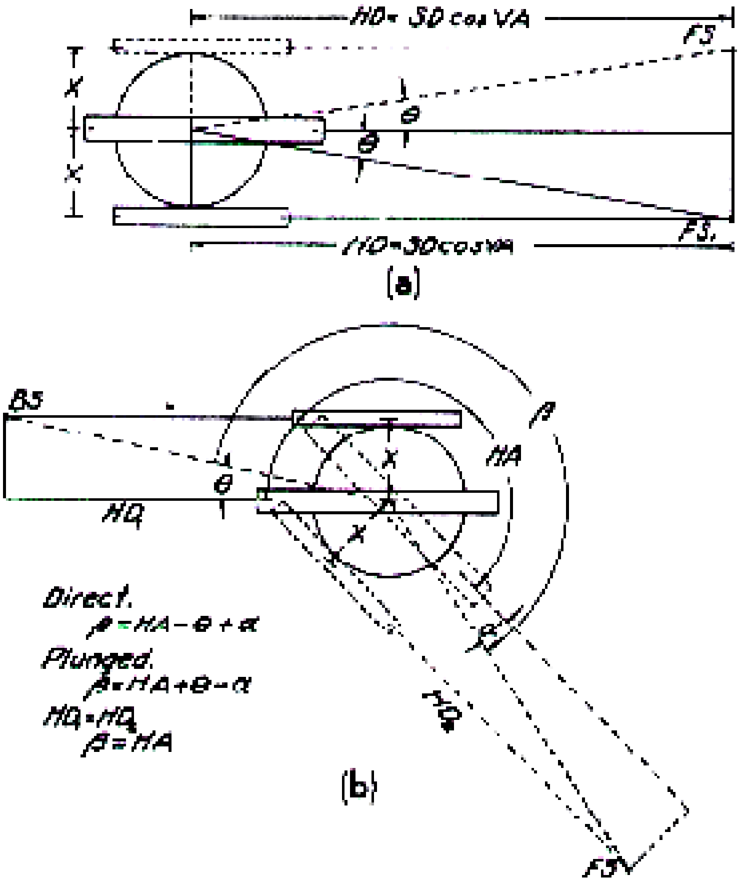


Fig 2.3 Use of side telescope when in adjustment.

In selecting stations, use as few as possible, since setups in steeply inclined openings are tedious and time-consuming at best. The back-sight is very apt to be short and the foresight long on starting through the incline. Some engineers prefer to place their backsight station at a distance equal to the foresight. (It must be understood that the distances referred to are the horizontal ones.) By doing this no calculations for the eccentric angle are necessary, for the correction on the backsight cancels the one on the foresight. But this procedure is not recommended, for it saves office time at the expense of underground time.

In Fig. 2.3a, the eccentric angle is represented by θ. Knowing *x* and the horizontal distance, this angle is quickly found. Depending on the position of the side telescope, it must be added to or subtracted from the horizontal angle read on the plate. We see that, in the direct position, FS_1 is in line with the side telescope. The main telescope is aligned with a point the distance *x* to the left of FS_1. The angle read on the vernier corresponds to the direction of the main telescope. Therefore, the angle θ must be added to the plate reading in order to obtain the true horizontal angle. In the plunged position the reverse is true. Here,

$$\theta = \tan^{-1} = x / HD,$$

HA = plate reading + θ, when the side telescope is on the right side (direct reading),

HA = plate reading - θ, when the side telescope is on the left side (plunged reading).

It is advisable to emphasize that the horizontal distance and not the slope distance must be used in calculating θ.

This analysis explains the effect of the eccentric angle. If one of the pointing, BS or FS, is taken with the main telescope, only one calculation for θ is necessary. This should be kept in mind. Very frequently it happens that the backsight on starting the survey can be taken with the main telescope. On arriving at the last inclined foresight station, the next foresight can also be taken with the main telescope. This procedure is recommended not because it saves office calculations but because it expedites the underground operations. The notes must definitely indicate such manipulations.

Values for θ are usually quite small (that is, minutes and not degrees). Their amount depends on the horizontal distance. For example:

assume x = 0.30 ft. and HD = 50 ft.

$$\theta = \tan^{-1} = 0.30/50 = 21' \text{ approx..}$$

Figure 2.3b shows the use of the side telescope on both BS and FS. Here θ equals the BS correction and α the FS correction.

The true horizontal angle is β; and the angle read on the plate is HA. The dotted outline gives the direct position when shooting the FS. Thus,

telescope direct, β = HA - θ + α,

telescope plunged, β = HA + θ - α,

If $HD_1 = HD_2$, then $\theta = \alpha$, and the plate reading is the true one (β = HA).

Example 2.1: From the following notes calculate the corrected angle right. The side telescope is on the right-hand side when the main telescope is in the direct position.

x	IS	HA	VA	SD	HD	θ	α	$HA_1 = \beta$	FS
0.30	B		0º 0'	30.00	30.00	0º34'22"			A
0.30	B	172º35'	-70º20'	120.00	40.39		0º25'32"	172º26'	C

Note: foresight station A is actually the backsight station.

log cos 70º 20' = 9.5270463

log 120.00 = 2.0791812

log HD = 1.6062275

HD = 40.39 ft.

$\theta = \tan^{-1} = 0.30 / 30$

log 0.30 = 9.4771213

log 30 = 1.4771213

log tan θ = 8.000000

θ = 0º 34' 22"

$\alpha = \tan^{-1} = 0.30 / 40.39$

log 0.30 = 9.4771213

log 40.39 = 1.6062275

log tan α = 7.8708938

α = 0º 25' 32"

β = 172° 35' - 0° 34' 22" + 0° 25' 32" = 172° 26' 10" = 172° 26'.

To adjust for faulty standards and vertical collimation, as well as to double-check, take two readings, a direct and a plunged reading. The average of the two gives the correct angle.

2.4.2. Modified Adjusted Method

For want of a better name, this complex designation is given to the method that follows. Strictly speaking, the procedure is not done with aligned telescopes. It is, however, more closely related to the adjusted method than to any other. Instead of aligning the two telescopes, determine *x* and the angle of divergence or convergence. Knowing this angle, you can readily correct for HA and θ *or* α.

The value of *x* can usually be safely assumed as constant after it is once determined. A radical change in the position of the vertical cross hairs may necessitate checking.

To determine θ and α, follow the instruction already given. To the values thus obtained, add or subtract the angle of divergence or convergence ϕ. Figure 2.4 illustrates the four possibilities. Referring to Fig. 2.4a, we can separate the diagram into two parts. A rectangle with ends x and sides HD constructs one figure, and a triangle aFSb the other. The angle ϕ is in this case diverging. The corrected angle β = HA + θ + ϕ. The following example illustrates the calculation for Fig. 2.4a.

Take x = 0.30ft ; bFS = 0.10ft.; HD = 50ft.; HA = 196° 28'.

$\theta = \tan^{-1} = x / HD = 0.30/50 = 0° 21'$ approx.,

$\phi = \tan^{-1} = 0.1/50 = 0° 07'$ approx.,

$\beta = 196° 28' + 0° 21' + 0° 07' = 196° 56'$.

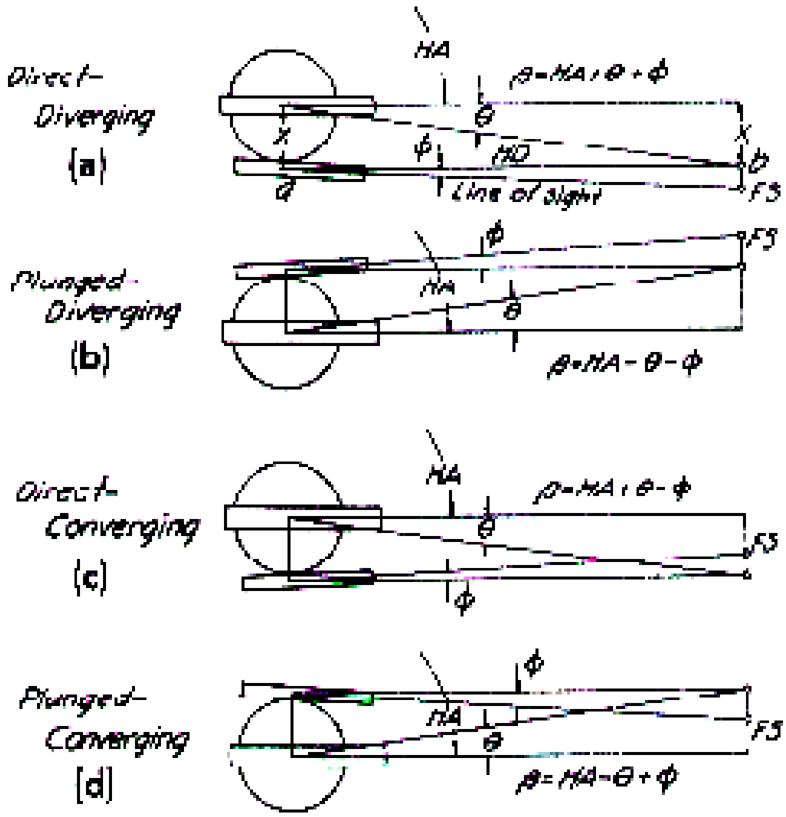


Fig 2.4 Out of adjustment method for the side telescope.

Figure 2.4 illustrates the analysis for foresight stations. The same type of correction must be applied to the backsight observation if not taken with the main telescope. Note that the HD is used.

2.4.3. Out of Adjustment Method.

This is the simplest and most reasonable procedure. It may become cumbersome when attention must be given to placing a shaft or raise on a definite line. With such work the adjusted method is easier to apply. On the other hand, if it is necessary to carry the meridian only through an inclined opening, then the out of adjustment method is preferable.

The operation is based on the fact that when the telescope is plunged, all common mistadjustment of the transit except plate levels are compensated for. There is no need of finding *x*, or any eccentric angle. If the side telescope is not shifted between the direct and inverted observations, it can only be the same distance on each side of the center of the instrument for each operation. The average of the two plate readings must then give the angle through the center of the instrument, this angle being the true one, compensated for collimation, standards, and eccentricity of the telescopes. This operation may be likened to the adjustment of the vertical cross hair for collimation. It will be recalled that this adjustment is based on the fact that two points are established, one direct and one plunged. The cross hair is then set on the point halfway between the two. Figure 2.5 illustrates the principle. The direct and plunged operations are shown separately, but the reader should visualize them as superimposed. To simplify things further, the BS was taken with the main telescope. The same argument holds true with the BS as with the FS.

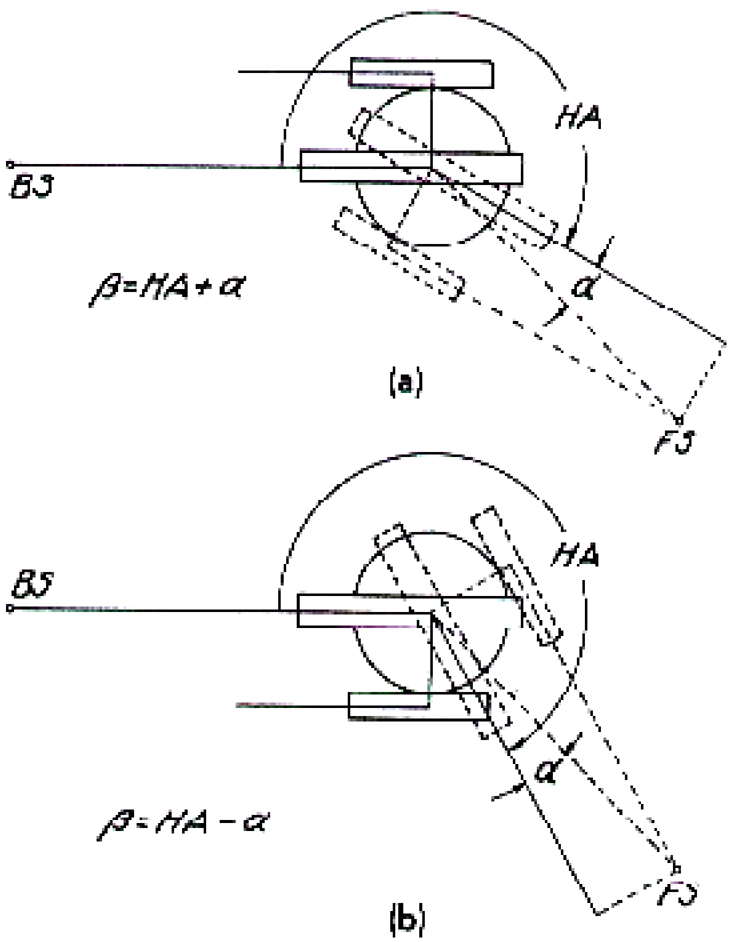


Fig 2.5 Use of side telescope when in adjustment

In Fig. 2.5a, the correct angle $= \beta = HA + \alpha$. In Fig. 2.5b, the correct angle $= \beta = HA_1 - \alpha$. Note that the eccentric angle is α in each case. This is true because the distance *x* cannot have altered and the FS has remained in the same position. Similarly, the true angle β is the same in each instance; only the plate angle changes. Therefore

$$\beta + \beta = (HA + \alpha) + (HA_1 - \alpha),$$

$$2\beta = HA + HA_1,$$

$$\beta = (HA + HA_1) / 2$$

From this simple expression we see that it is necessary only to read the HA direct and plunged, add them together, and divide by two. The result is the correct horizontal angle. There is no determining constants, adjusting the transit, checking its adjustment underground, or introducing possible calculation errors.

Once in a great while it happens that this method must be combined with the modified adjusted method. The reason for this results from difficult setups where projecting rock or timber prevents either a direct or a plunged shot. Only in cases of emergency, when approximate data are wanted at once, should this be done. Since instrumental inequalities are not compensated for, the azimuth is very apt to be off unless the transit is in first-class adjustment.

As with the previous method, the true angle is directly read when BS = FS. This holds whether the main telescope or the side telescope is used.

The best technique for measuring the angles is as follows. It can be modified, but it has been found, at least for the beginner, that less confusion will creep in if these instructions are obeyed.

The side telescope is attached and the instrument set up. Set on zero, and take a backsight with the telescope in the direct position. (Depending on conditions, use either the main or the side telescope.) With the upper motion released, observe the FS. Read both the HA and the VA. Without plunging, the angle is doubled. Do this simply as a check on the first operation; for in making steep sights the wrong tangent screw may be moved, and doubling the angle will immediately indicate this mistake. Read the VA again. Now set on zero, plunge the telescope, and take another BS. Release the upper motion and observe the FS. Read the HA and VA. Double this horizontal angle for a check. The average of all four VA gives the true VA, and the average of the direct and plunged HA gives the true HA. When a person is sure of himself, only one reading in each case need be taken with but one setting on zero.

The following example illustrates the calculations involved.

From the following notes find the correct HA and VA.

		Measured angle		Corrected angle			
BS	IS	HA	VA	HA	VA	Telescope	FS
20	21	162º 17'	- 68º 21'			Direct	22
		324º 35'	- 68º 21'				
		162º 58'	- 68º 27'			Plunged	
		325º 55'	- 68º 27'				
20	21			162º37'30"	-68º24'	Average	22

324º 35' ÷ 2 = 162º 17' 30",

325º 55' ÷ 2 = 162º 57' 30",

(162º 17' 30" + 162º 57' 30") / 2 = 162º 37' 30",

(68º 21' + 68º 27') / 2 = - 68º 24'

Attention is further called to this method in that no measurements with the tape are necessary for transferring the meridian. In the first method discussed, the SD had to be known, and a mistake in measuring the distance or calculating the HD would, of course, throw the azimuth off.

2.5. Top Telescope

Use of the top telescope is, in most ways, similar to that of the side telescope, but it is inferior in one important respect: it is impossible to double the horizontal angle and compensate for instrumental adjustments and get a plunged reading of the vertical angle. We are unable, therefore, to eliminate misadjustments of the transit a very important factor for steep shots. By careful adjustment, by the use of a striding level, and by the determination of certain constants for the deviation of the two lines of sight, accurate work may be performed with the top telescope. However, the top telescope is not important enough, in transferring the meridian, to devote space and time to the analysis of these refinements. For the alignment of headings with line plugs, month by month, the check on ordinary adjustments is satisfactory. As the heading advances, the correct azimuth should be transferred to the breast from time to time with the side telescope.

The adjustments usually performed are to put the vertical cross hairs in the same plane and to make the lines of sight through the horizontal hairs parallel. Before doing this, correct the collimation and the index error for the main telescope and check the standards carefully.

Even though extreme care is exercised, it is doubtful if a nearly perfect adjustment can be made. There is no doubt that it can be sufficiently approached for ordinary control of directions. After the auxiliary is mounted, sight the main telescope cross hair on a distant plumb bob string. With the adjustment attachment, bring the vertical hair in the top telescope in line. The constant *x* is found as shown in Fig. 2.6a. This is done in exactly the same way as for the side telescope. If parallelism is wanted, the horizontal hair is moved until *C* equals *x*.

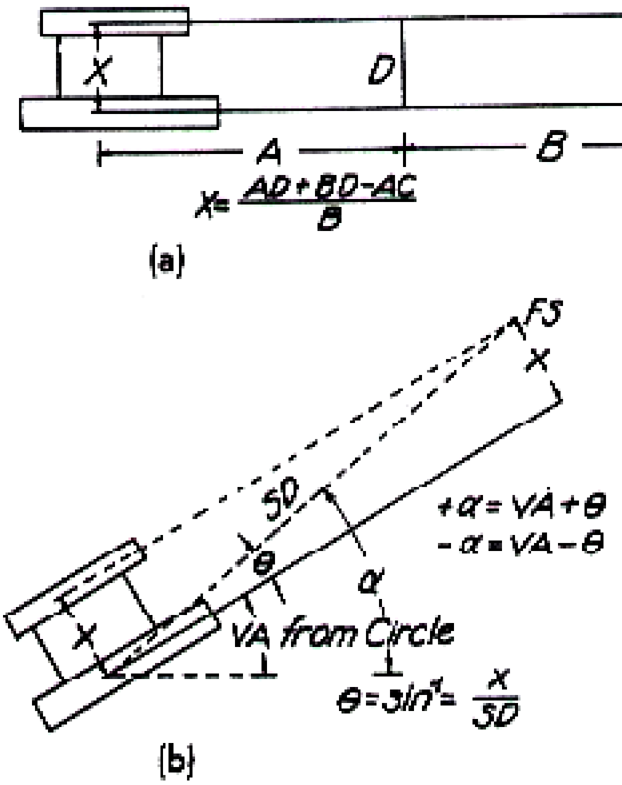


Fig 2.6 Adjustment and use of the top telescope.

It is convenient to apply the diverging or converging angle ϕ instead of altering the position of the cross hairs. Where both side and top telescope must be used, it would be desirable to align the top telescope hairs and use the out of adjustment method for the side telescope.

Figure 2.6b shows diagrammatically the measurement of a vertical angle. It is important to note that θ is a function of the sine, and not the tangent as was the case with the side telescope. Also the SD is used here and not the HD.

The HA is read directly without correction. Because of slight misadjustments in the instrument, the precision obtained is doubtful.

Since there is no way to plunge the telescope, accuracy must depend on the perfection of the instrument. If an index error is present and its amount is unknown, the vertical angle will be incorrect, and again there is no compensation resulting through inversion. The following example illustrates the calculations for a top telescope.

From the data given, determine the true VA, the eccentric distance *x*, and the HD. (The lines of sight are adjusted before making the survey.)

A = 100 ft. B = 75 ft. C = 0.48 ft. D = 0.42 ft.

x	IS	VA read on circle	SD	HD	Eccentric angle, θ	True VA, α	FS
0.34	A	+ 72º 36'	220.00	65.48	0º 05'	+ 72º 40'	B

$x = (AD + BD – AC) / B$

$= (100 \times 0.42 + 75 \times 0.42 - 100 \times 0.48) / 75 = 0.34$ ft.,

$\theta = \sin^{-1} = x/SD = 0.34/220.00 = 0^\circ 05'$,

$\alpha =$ true VA $= + 72^\circ 35' + 0^\circ 05' = + 72^\circ 40'$,

HD $= 220.00 \times \cos 72^\circ 40' = 65.48$ ft.

2.6. Summary

In concluding the discussion of auxiliary telescopes, the advantages and disadvantages of each may be summarized as follows.

(a)Side Telescope. Advantages:

1. Through direct and plunged readings, misadjustments of both the side telescope and the transit, with the exception of plate levels, verticality of the vertical axis, and eccentricity in the plate, are eliminated. The last two are probably of no consequence with the modern transit. The levels are readily adjusted for all practical purposes.
2. Vertical angles are read directly, and the average of direct and inverted readings gives the true angle without recourse to correcting for index error.
3. Horizontal angles are readily found without aligning the telescope.

Disadvantages:

1. For certain types of control work, the horizontal angle cannot be read directly. This can be handled by calculating the offset (*C* or *x)* to put the station on line.

(b)Top Telescope. Advantages:

1. Horizontal angles are read directly. For control this is satisfactory. For azimuth the results are doubtful unless the instrument is in perfect adjustment.

Disadvantages:

1. Lacks advantages 1 and 2 given for the side telescope.
2. The vertical angle cannot be read directly for control. Offsets may be calculated (*C* or *x*), and the inclination may be put on line.

When the use of the auxiliary telescope is contemplated, the plate bubbles must be carefully adjusted. If the top telescope is to be used, the standards also must be checked, since they are probably the chief source of error.

Many devices have been suggested for eliminating the eccentricity of the telescopes (such as FS or BS targets with an offset equal to *x).* Unless the use of the auxiliary telescope has become daily routine instead of occasional, their applicability is doubtful, since the telescopes must be in adjustment.

Setting up the transit in openings inclined more steeply than 60° is often very difficult and time-consuming. For this reason apparatus has been evolved to take the place of the tripod. The two forms most commonly used are a bracket screwed or bolted to the timber, with a tripod head attached to the free end, and an extension bar. The bar has the advantage of requiring no bolt holes or tools to set it up. It may also be used in ordinary traversing in a drift when it is desirable to place the transit above and out of the way of haulage operations.

2.7. Transferring the Meridian without Auxiliary Telescope

It sometimes happens that the engineer is called upon to carry a survey through steeply inclined openings with a transit lacking a side or top telescope. There are several methods of doing this, though they give as a rule only reasonably approximate results. If considerable time is devoted to the work, and the results of a number of setups are taken, fairly close work may result. Approximate methods will not ordinarily have to be used around an operating mine, since the engineer will be provided with the correct equipment. However, occasionally the engineer is called upon to survey a temporarily closed mine. He may be provided with an ordinary transit and on arriving at the property may learn that an auxiliary telescope is needed. In such cases the following methods will prove useful.

Figure 2.7 illustrates a method yielding approximate results for transferring the azimuth, elevations, and coordinates down an inclined opening. A wire or heavy cord is stretched between stations *C* and *H*.

(a)Azimuth

To determine the azimuth of *CH*, plumb bobs are hung on the wire at *D*, *E*, *F*, and *G* (the resulting sag as indicated in the sketch is of no consequence). If the shaft is windy and difficulty occurs in steadying the bobs, heavier weights on piano wire may have to be substituted for ordinary plumb bobs. If the utmost accuracy is desired, the plumb bob wires should all hang from the same side of the supporting wire. After coplaning or triangulating on *DE,* stations A and B are set on line. If the transit is in first-class adjustment, this can be done very accurately. Stations *A* and *B* are tied into the main survey, and the coordinates and elevations of *B* are transferred to *C*. If the wire *CH* encounters no obstructions, the upper bobs *DE* and lower bobs *FG* will be in the same plane.

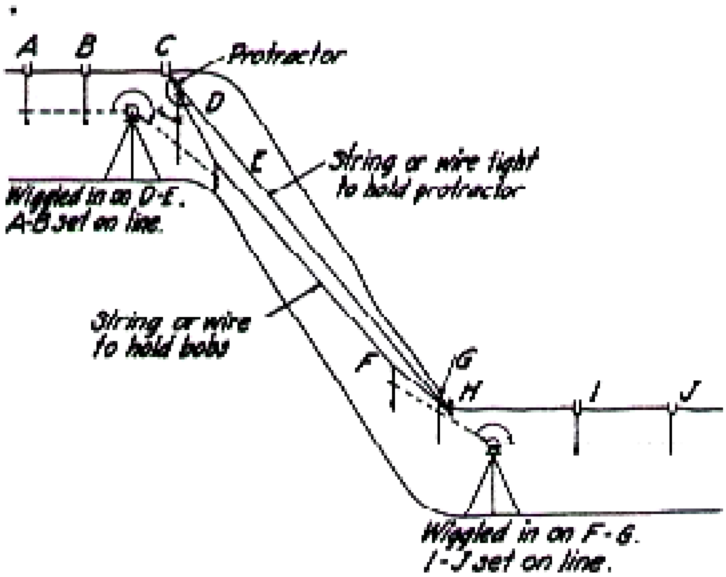


Fig 2.7 Surveying a steep raise without auxiliary telescope

At the bottom, the operation with the transit is repeated and stations *I* and *J* are set on line. The coordinates and elevations of *H* are carried to *I*. Using *J* as an instrument station, the lower survey is started.

Under certain conditions of inclination and cross-sectional size of shaft, this operation may be simplified. Stations C and *D* may coincide, as may *G* and *H.* The traverse is then run to *C***,** and *C* is occupied with *B* as BS and *E* as FS. Below, *H* is occupied, *F* is used as a BS, and/is used as the FS. The wiggling in is thus eliminated. The vertical angle is found as suggested later. Rights and lefts and ups and downs may be taken from the taut wire to plot the profile of the incline.

(b)Elevations and Coordinates

To carry the elevation and coordinates down the incline, the wire is tightly retretched from *C to H.* A lightweight protractor is hung as near *C* as possible, and the inclination is read. This is repeated at *H*, and the two are averaged.

If more accuracy than the 30 min. reading of the protractor is desired, the following alternative procedure is used: hang a plumb bob from *C, so* that it will fall somewhat below the horizontal line of sight through the telescope. Spot the horizontal line of sight on the string and measure the vertical distance to the wire. Where the line of sight intersects the wire, locate a point. Then measure the horizontal distance from the plumb bob string to the wire. We now have two legs of a right triangle, and the vertical angle can be calculated. If the transit is located too near *C* for focusing, an ordinary carpenter's level may be used to form the triangle. Measure the SD by stretching the tape from *C* to *H.* If this distance exceeds the tape length, either fasten two tapes together or measure to an intermediate point on the tight wire. Figure 2.8 shows the use of an inclined or tilted transit.

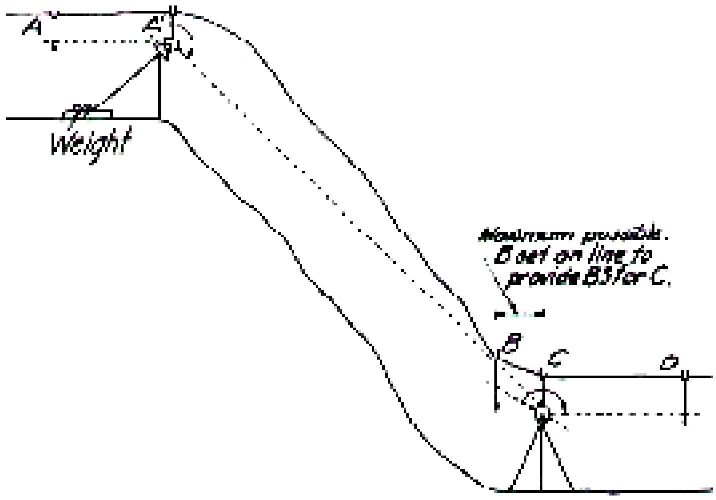


Fig 2.8 Inclined transit shot

It is very necessary that the horizontal axis of the telescope (the bubble perpendicular to the line of sight) be accurately adjusted. The instrument must be shifted about until *C* can be established after backsighting *A* with an inverted telescope, then plunged, and *C* sighted, always with a centered bubble. This places *A, A*' and *C* in the same plane (assuming perfect adjustment). Station *B* is then set on line. The two front legs of the tripod are shortened until *C* can be seen. A long plumb bob string placing the bob near the floor may be required.

The back leg is held in place either with a heavy weight or by clamping to a plank. The distance *BC* should be the maximum possible in order to avoid a short BS when at *C*. The same is true of *AA′*. The position of station *A′* may be established with the transit roughly set up. Then, after removing the instrument, place a stull across the opening so that it will be above the centering mark with the telescope level. The transit is then set up in the former position and is carefully leveled, the telescope is made level, and by means of a plumb bob *A′* is set above the centering mark. The telescope is now plunged, the lower motion is clamped, and the plate is revolved with the upper motion until the bubble is exactly centered. Station *A* is located. With the telescope plunged back to the direct position, station *C* is established. Neither the upper nor the lower motion must be touched. Next *B* is set on line. If a so-called "stovepipe" prismatic eyepiece is at hand, station *B* will not be needed, since *C* can be occupied and the BS taken directly on *A′*.

By also observing *A* with the telescope in the direct position and plunging to sight *C*, collimation and standard adjustment are compensated for. The midpoint between the two points is taken as the location of *C*. With *C* as the FS, *B* is set on line.

The SD is measured from the instrument to *C*. An HI at *A′* and an HS at *C* with the vertical angle gives the necessary data for transferring the elevation.

Station *A* is tied into the main traverse, it is occupied, and *A′* is located. The vertical angle is measured by leveling up the telescope and reading the vertical circle. Station *C* is then observed and the angle read. The difference between the two gives the vertical angle.

Modifications of these two methods may be made to give results from very rough to fairly accurate work.

CHAPTER – 3
TRANSFERRING THE MERDIAN
VERTICAL OPENINGS (SHAFT PLUMBING)

3.1. General

Probably mines through a vertical shaft, and the underground workings are connected by vertical raises or winzes. Transferring the azimuth through such an opening is one of the most important duties of the engineer. The care and precision with which this must be done depend on the extent of the underground workings and their connection with each other in a vertical plane. Relatively shallow mines having flat ore bodies restricted to more or less equidimensional areas will not ordinarily require the same attention as narrow, deep, long vein structures.

The same basic principles are used in all cases, the technique being altered to suit the needs of each case. Occasionally, a simple magnetic bearing taken with the transit compass or a Brunton proves entirely sufficient. At other times the alignment of two wires with a third point by the unaided eye may be used. Ordinarily, however, a transit is required.

The equipment for shaft plumbing has shown little change in many years. Because of shaft deepness, some engineers have tightened up the procedures, but a surprisingly small number have adopted the optical transit, even for the more precise work. The most notable improvement is the almost universal change to triangulation (Weisbach method); 25 years ago coplaning predominated. And because many mine workings are now connected with two shafts, the two-shaft method is becoming more feasible. The most important recent innovations are given. The application of gyroscopic equipment is receiving more attention.

There is still much reluctance to observe the plumb bobs in motion, but several surveyors use this procedure. Two main methods of transference are used: the one-shaft method and the two-shaft method. Wherever possible, the latter is preferred.

3.2. Equipment for Plumbing

The equipment for shaft plumbing has occasioned considerable discussion. Of such equipment as reels, wire-centering devices, screw shifters, shapes of plumb bob, bronze or steel wire, links in the wire at the sighting end, type of transit, immersion liquids, vanes on the bob, etc., the more important will be briefly discussed. Under particularly adverse conditions knowledge of them may prove of considerable value.

(a) Reels

A reel is necessary for preserving, lowering, and winding up the wire. Without a substantial reel it is difficult with the bare hands to handle fine piano wire holding a light weight; it is impossible with a massive bob. The reel may be made with a base which can be clamped, screwed, or bolted in place. Many times the reel and the screw shifter are combined.

(b) Wire-centering Apparatus

This device is used to hold the wire in position after the center of swing has been determined. Some engineers prefer, after finding the center of swing, to clamp the wire in a stationary position before sighting. If shaft plumbing is limited to a maximum of 200 to 300 ft. intervals, clamping probably will not be necessary except under the most trying conditions of air currents and falling water. A 500 lb. bob on 0.1 in. diameter wire is said to remain perfectly quiet.

(c) Screw Shifters

The screw shifter is used to shift one wire into the plane of the transit and the other wire at the starting station (or both wires may be shifted back and forth). It can also be used to move the wire in prearranged directions to ascertain whether or 'not the wire is "hung up" on obstructions in the shaft, although its usefulness for this is questioned. For example, the upper end is moved; say 2 in. west, 3 in. north, 3 in. east, etc. If the wire is free, its lower end should more or less follow these movements; an observer on the lower end does the checking. This.

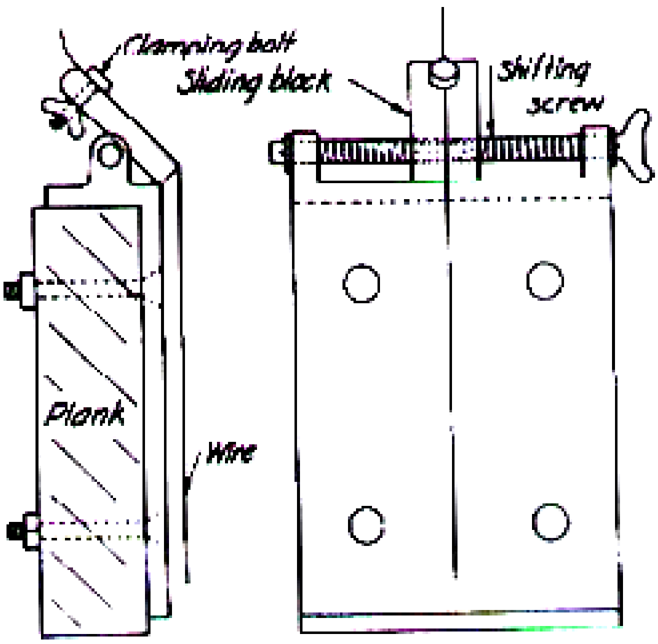


Fig 3.1 Screw shifter

manipulation and method of shifting wires into line have never been practiced much . Figure 3.1 illustrates a very simple form of screw shifter.

(d) Plumb Bobs

Lead, brass, and steel plumb bobs have been tried. It has been thought that magnetic attraction influences steel bobs and steel wire. This might well be so in mines containing strongly magnetic material (magnetite, ilmenite, pyrrhotite). Pipes and electrical conduits have been blamed for affecting the bobs. With a few exceptions it probably makes little difference, which are used. Figure 3.2 shows a common type of bob, made of steel shafting and weighing about 30 lb. It may be a pipe filled or partly filled with lead or a brass or bronze bushing filled with sand.

A large bob is easily made from lead-filled pipe. The pipe is turned down on a lathe to provide a uniform, smooth, concentric surface. A small axial passage is left through the lead filling for the wire to go through and fasten at the bottom. A 3-in.-deep depression is turned out at the top, and the rim of the pipe is shaped to a knife edge. Dripping water has least effect on this top. In a test a 75 lb. bob was hung in a large, uncovered drum to prevent agitation by air currents. On 4000 ft. of wire, the bob required a quarter-hour to steady to a total swing of 2 in. Time required to complete the swing was 70-80 sec.

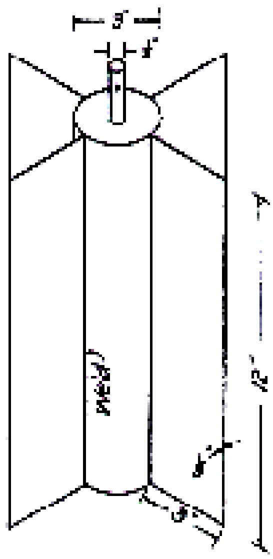


Fig 3.2 Plumb bob for shaft plumbing

For designing a lead-and-pipe plumb bob, the following formula is useful:

$$x = \frac{W}{(3.78\, d^2) + w}$$

where

x = length of bob, feet (i.e., the length of pipe if filled with lead),

w = weight of pipe, lb/ft,

W = weight of bob desired. Ib.,

d = inside diameter of pipe, in.,

710 Ib. per cu.ft. used for lead in the formula.

The attaching hook must be accurately centered; otherwise the bob revolves with an eccentric motion which prevents it from coming to rest or deflects the wire from a vertical position. Some have claimed that the wings are of slight help; their function is to dampen the vibrations and the swinging and whirling of the bob. The size and weight of the bob needed depend on the air velocity and the quantity of water falling in the shaft. Fifty-pound bobs are usually heavy enough, and under favorable conditions much lighter ones will do. But bobs weighing 110 lb. have been used.

(e) Wire

Steel wire is ordinarily used. Under exceptional magnetic conditions bronze may be required. (A suitable phosphor-bronze wire would be 0.080 in. diameter; tensile strength, 140,000 psi; ultimate strength, 704 lb.) A very fine wire is necessary. No. 12 ½ piano wire (0.03 in. diameter) is commonly employed. This wire will support a 60 lb. bob with plenty of safety allowance. If lighter bobs are satisfactory, a smaller wire can be used. Open flames must not touch the wire, especially when under the strain of the plumb bob, since only a little heat snaps the wire, making it fly rapidly upward to say nothing of losing the plumb bob in the sump.

No. 25 armature winding wire (equivalent to no. 12 ½ piano wire) has also proved suitable (this is a moderately strong, tinned steel wire). Piano wire is sold in coils. By dressing or straightening the wire, spinning of the bobs is reduced. The tendency to spin results from an inherent twist given to the wire during drawing.

A few sizes are given in Table 3.1. With the exception of the piano wire numbers, * gauge sizes of the wires are not included because of the lack of uniformity between the various standards. Attention is called to the tensile strength, which varies with the diameter of the wire. These data are for the best grade of music wire.

(* Personal communication from Mr. Charles Vavra, Manager of Piano Supplies Deprt., Steinway and Sons, Long Island City, N.Y.)

Table 3.1 Strength of Music Wire

Piano wire number	Diameter, inches	Area Sq.in.	Tensile strength psi	Ultimate strength lb	Weight of plumb bob (factor of safety of 4) lb
1	0.010	0.00007854	390,000	31	8
5	0.014				
8	0.020	0.0003142	351,000	110	27
10	0.024				
	0.025	0.0004909	340,000	167	42
11	0.026				
12 ½	0.030	0.0007069	332,000	235	60
15	0.035	0.0009621	325,000	313	80
17 ½	0.040	0.001251	317,000	397	100
20	0.045	0.001590	310,000	493	123
	0.050	0.001964	305,000	600	150
	0.060	0.002827	295,000	834	210
	0.070	0.003848	290,000	1116	280
	0.080	0.005027	282,000	1418	350
	0.090	0.006362	275,000	1750	440
	0.100	0.007854	270,000	2121	530

3.3. Preparation for Plumbing

3.3.1. Steadying the Bob

For dampening the movement of the plumb bob, water, molasses, very viscous oils, and other liquids have been used or suggested. Some engineers pour a layer of oil on water in order to reduce the effect of falling water. The beneficial effect of wings on the plumb bob is debatable. The most satisfactory thing is to shut off air currents and falling water as far as possible and brattice up connections to the shaft. By confining the plumbing to distances not exceeding 200 to 300 ft., steadier conditions result and the bob will come approximately to rest in a shorter period.

When a cover is used on the container (usually an oil drum), a hole in the cover ranging from 2 in. to 6 in. in diameter is sufficient for passage of the wire. The hole is connected to the edge of the cover by a slot.

Many engineers prefer to suspend the bob in a glass vessel. This enables contact between the bob and the bottom to be detected. If glass is not used, due precautions must be taken to ascertain that the bob is swinging free. One way is to release a small coil of wire wrapped around the plumb wire and see if it reaches the bottom.

3.3.2. Displacement of Plumb Bob

Throughout the literature on shaft plumbing there is speculation on what causes the difference in distance in the wires at the supported end and at the plumb bob. Many opinions are given concerning the effect of nearby or distant masses on the displacement of the wires.

Responsibility for the distortion of the wire plane has been assigned to many sources: weight of the plumb bob, weights used for maintaining tension on rope guides, electrical conduits, cables suspended in the shaft, nearby pumping equipment, masses of ore (if magnetite or pyrrhotite, there is a magnetic effect in addition to density), open spaces (shaft, slopes), and so on.

However, the wire plane may be displaced in four special situations: (1) when heterogenous strata or rocks of differing specific gravity surround the shaft; (2) when there are large voids in the rock mass, of which the shaft is one (the gravitational effect of a void is negative); (3) when there are drifts and shaft stations joining the shaft (inset roadways); and (4) when the shaft equipment is of high density and concentrated near the plumb bobs.

Air currents and falling water are responsible for most displacement troubles. Often failure to establish the zero point of the swinging bobs contributes to the distortion of the wire plane.

3.3.3. Ascertaining Mean Position of Wire

Quite often the plumb bobs will not come to rest within the time available. In such cases the mean position must be determined. (Quite acceptable work has been done by observing moving wires) Considerable difference of opinion exists as to the minimum number of swings and the best method of counting the swings. Under the existing conditions the plumb bob does not act like a simple pendulum but has an oscillating motion. Some engineers simply bisect the swing, which appears normal to the line of sight (transit to wire). Unless the displacement is slight, this is probably not satisfactory. Simultaneous counting of the swings at right angles to each other generally gives the best results. This is done by mounting a scale, as shown in Fig. 3.3; about 3 in. back of the wire. For the highest accuracy, low-power reading telescopes may be used. For less important work the swings are safely counted with the naked eye. The scales should be firmly attached to timber that will not be disturbed until the work is finished. A double scale is convenient because both the azimuth plane and the distance between the wires are required. The most important is the azimuth plane.

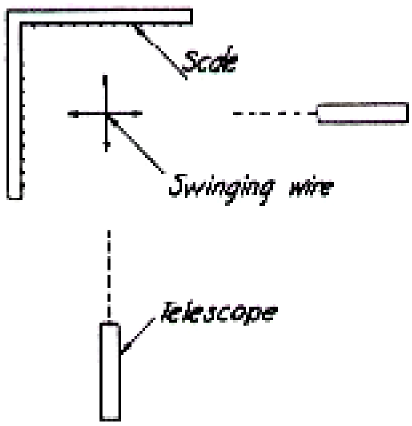


Fig 3.3. Finding the rest point of a swinging wire

Several additional procedures have been advanced for accurately finding the rest or zero point of the swing. It is customary to use the arithmetic average to find the mean position of the oscillations. This may result from averaging a number of extreme swings observed as the wire is moving parallel to the scale, or a series of two extremes (right and left) are individually averaged and the mean of these averages is found. Several engineers recommend that the bob deliberately be forced to swing parallel to the scale or perpendicular to the line of sight by giving it a slight push.

3.4. One-Shaft Method

There are three modes of attack when only one shaft is available: coplaning (also known as wiggling or jiggling in), triangulation or Weisbach method, and special cases of these. The procedure of hanging the wires and steadying them is identical in all instances; this applies to the two-shaft method as well.

In the past, use of the two methods was about equally divided. In later years, a decided change to triangulation has taken place, although many engineers assert that coplaning may be applied under conditions where it is impossible to use triangulation. The author has difficulty in understanding how this can be. Triangulation, if conducted in the approved way, is practically coplaning, since the transit station is but a few minutes to one side of the plane through the wires. There is apparently little difference in the precision obtainable by the two methods. Prejudice, time available and labor, appear to influence the selection. With reasonable care and use of the ordinary 1 min. transit, the transference of the meridian with an error not exceeding 1 in 10,000 may be expected by either method.

Many of the references given herein cite precision, in excess of 1 in 30,000. In order to obtain as much precision as possible, the shaft should generally be plumbed in sections of 200 to 300 ft. at a time (under exceptionally favorable conditions this may be extended to several thousand feet). That is, the wires are rehung or the mean position found and clamped at the lower stations before proceeding to lower levels. This removes excessive swinging and uncertainty as to the center of swing, and lessens the likelihood that the wires will be in contact with the walls of the shaft.

The distance between the wires is measured at the top, where they are supported, and again at the place where the meridian is taken off. These two measurements should agree very closely. It is impossible to make a definite statement regarding the magnitude of the difference. The amount of falling water and the effect of air currents is too uncertain. Possibly it may be said that if the disagreement is more than 0.02 ft. the cause should be ascertained and eliminated, or an adjustment made for the effect. Air currents will probably be responsible. If there were no doubt that the lack of conformity was confined to the plane of the wires (interval too long or too short), the difference would cause only a slight error in the coordinates when coplaning. However, as the distance between the wires is very frequently less than four feet, an error of several hundredths of a foot at right angles to the true wire plane causes too great an error in the azimuth.

For example, the distance between the wires is 4 ft.; one wire is 0.02 ft. out of the plane. The angular displacement is

$$\tan^{-1} \text{ or } \sin^{-1} = 0.02 / 4.00 = 17' \text{ approx.}$$

only 20 sec. can be permitted if 1:10,000 must be maintained. This indicates the importance of knowing the reason for a difference between the two measurements and of correcting the fault.

The distance between the wires is usually measured to the nearest thousandth of a foot. By so doing there is little uncertainty regarding the nearest hundredth.

3.4.1. Coplaning

This is also known as wiggling in or jiggling in. The objective sought is to place the transit exactly in the plane formed by the two wires. Coplaning is accomplished by moving the transit back and forth across the wire plane until the vertical cross hair is exactly on line.

Under this section only two wires (therefore, forming one plane) will be discussed. Later, the utilization of more than two wires will be taken up.

In order to avoid an adjustment in the traverse, and to save time, the transit should be in excellent adjustment. The plate levels and the vertical collimation should especially be attended to. As angular measurements are turned in practically a horizontal plane, the effect of the standards is not so important, although they should be checked. The reason for preferring the transit in adjustment is that on plunging the telescope and recoplaning, two centering positions for the station will occur if the instrument is out of adjustment. The mean of these is the correct location of the station.

In case the direct and indirect operations do not coincide, the station may be located in the following manner. The engineer must have with him a block of lead about 4 x 4 x 3 in. One surface of this is hammered smooth. After coplaning has been accomplished, the lead block is firmly imbedded beneath the transit. A plumb bob is now hung under the transit and the point is transferred to the smooth side of the lead. The angle between the plane and the first permanent station is turned and repeated as many times as desired. Now plunge the transit and again coplane. Transfer this station to the lead block and turn the angle to the permanent

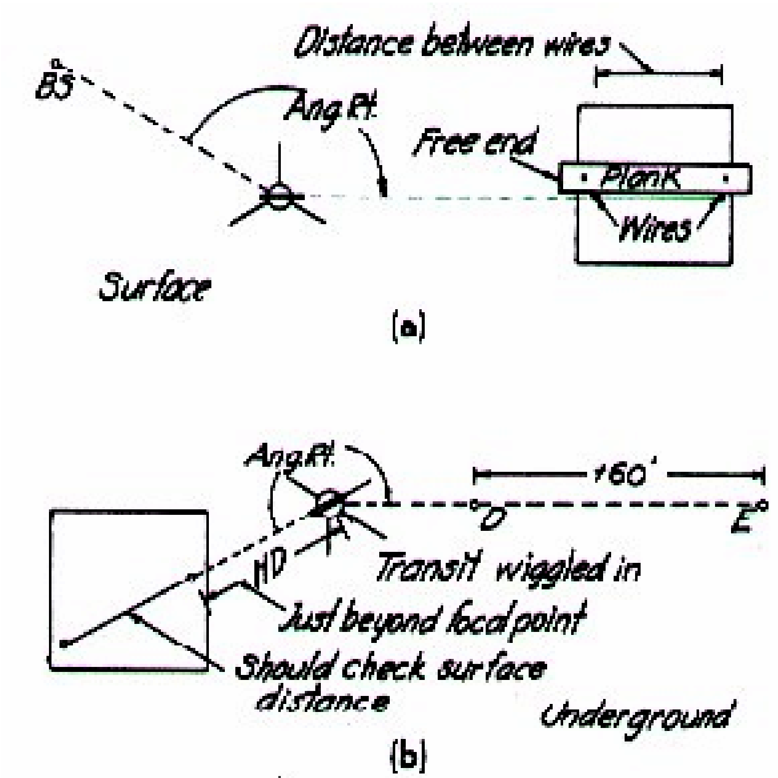


Fig 3.4 Shaft plumbing by coplaning (wiggling in)

station. The mean of the two sets of angles gives the true angle. For the transit station, the mean of the two points on the block is taken to provide a backsight. A pin can be stuck in the lead for the backsight.

To start the plumbing operations the survey system is carried to the shaft collar by traverse or triangulation. One end of a plank is nailed across the collar set (Fig. 3.4a) and the wires are hung in place. It is necessary, of course, before doing this to decide upon the most useful direction for the plank. The condition of the shaft for maximum wire interval and the convenience of the setup at the underground station must be investigated. The maximum wire interval should be used. But the effect of locating the wires in irregular positions with regard to the shaft walls should be considered. At the same time the direction of the wire plane should be such that the recommendations in Fig. 3.4b can be followed. The distance, instrument to *D*, should approach 60 ft. if possible. It is preferable to sacrifice a few inches in the wire interval in order to maintain this distance. A short backsight from *D* to the instrument station may involve an error equal to or even greater than that of the plumbing operation. Transferring the two points to the lead block involves a certain amount of error, as does centering the instrument at *D*. If the instrument station to *D* is short, the accumulated error may amount to several minutes.

The free end of the plank is lightly tacked in .place. A string is stretched along the two wires (see Fig. 3.4a). A stake is driven on line and a point marked on it. The transit is now set up and centered over this point. After the far wire has been sighted, the plank is moved, bringing the near wire into the plane. When coplaning has been accomplished, the angle from the last traverse station to the wire plane is turned. The telescope is now plunged and wiggled into the plane. Its new position is transferred to the stake. The mean of these two points is the instrument station. The closing angle is now turned from the traverse station to the new station. Horizontal or slope distance and vertical angles and HI are also taken in each case.

For the surface work, the distance between the near wire and the transit is not particularly important (15 or 20 ft. is satisfactory). As the instrument is usually above the wires, there is no difficulty in sighting each. Underground, where the wires as a rule extend through the line of sight, the location of the transit is of the utmost importance. If the transit is set too far away, the near wire will have to be moved in order to see the far wire; or chain links must be inserted. All of this bother can be avoided if the instrument distance is just beyond the focal distance of the transit, an interval which should be determined for the particular transit in use. When set just beyond the focal distance, the near wire can be entirely focused out of view and the far wire brought in. This obviates moving the wires or using links. It is convenient to attach a small piece of paper to one wire near the line of sight for aid in distinguishing the two wires. Figures 3.4b and 3.5 show this situation diagrammatically.

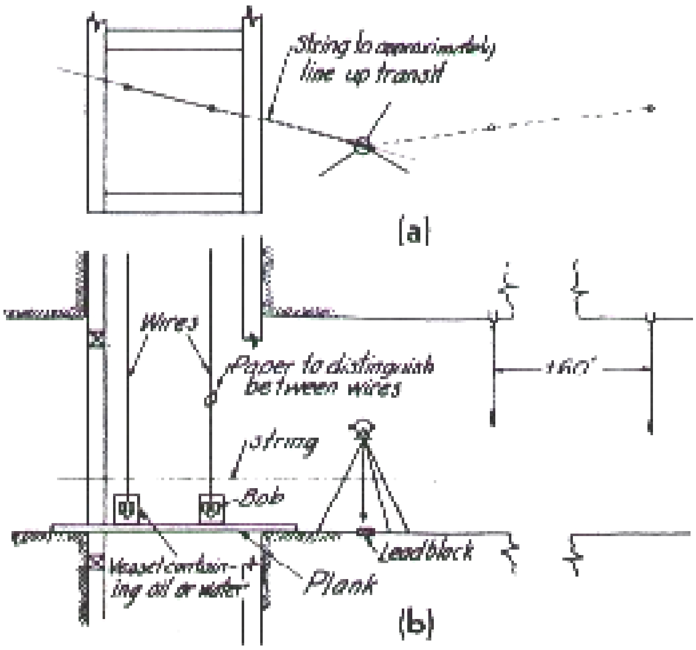


Fig 3.5 Coplaning on the underground setup

The following example illustrates the necessary calculations (Fig. 3.6 accompanies these notes).

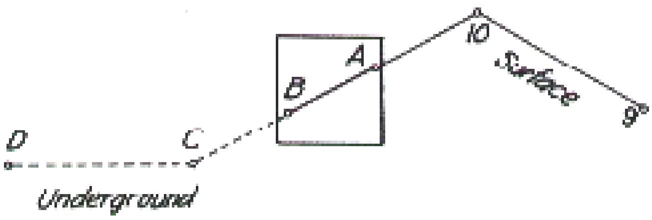


Fig 3.6 Example of coplaning

					Latitude		Departure		Coordinates		
BS	IS	Angle right	HD	Bearing	N	S	E	W	N	E	FS
8	9	230°	190.0	N50°W					6,000.00	4,000.00	10
9	10	130°	7.0	S80°W							*A*
10	*A*	180°	4.0	S80°W							*B*
A	*B*	180°	6.5	S80°W		3.04		17.23	5,996.96	3,982.77	C
B	*C*	215°	80.0	N65°W	33.81			72.51	6,030.77	3,910.26	*D*

Calculation of bearings.

360	260 = S 80° W
N 50 W	180
310	440
130	180

440		260	= S 80° W
180		215	
260	= S 80° W	475	
180		180	
440		295	= N65°W
180			

Total angle rights = 705°

310
705
1,015
720
295 = N 65° W, which checks.

There are three distances on the same line (10*A*; *AB*; and *BC*).

7.0 + 4.0 + 6.5 = 17.5 ft.

0.482708 = 3.04

log cos S 80° W = 9.239670

log 17.5 = 1.243038

log sin S 80° W = 9.993351

1.236389 = 17.23

1.529038 = 33.81

log cos N 65° W = 9.625948

log 80.0 = 1.903090

log sin N 65° W = 9.957276

1.860366 = 72.51

In the preceding example the distances were taken to the nearest foot and the angles to the nearest degree merely for convenience in illustrating the calculations. Data obtained in practice would not be so simplified distances would be the nearest 0.01 ft. and angles to possibly 10 sec. by repetition and averaging. The method used for recording the data is recommended. It is listed in an order which makes a continuous traverse of the shaft plumbing operation. Little opportunity is presented for confusion in the office. For the office record, additional columns would be available for recording elevation data and slope distances.

3.4.2. Triangulation (Two-Wire Method)

The two-wire method of finding the azimuth of the wire plane is frequently called the Weisbach method. If the angle measured by the instrument is small (from a few seconds to less than a degree), this term is correct; when the angle becomes very large (usually 60° is about the maximum), the Weisbach method is not being used.

The hanging and steadying of the wires is identical with the procedure in coplaning. All of the suggestions given there for selecting the suitable part of the shaft compartment should be followed. At the starting end of the wires either coplaning, by shifting the wire support, or triangulation may be followed. Under average conditions it may prove quicker to coplane.

Figure 3.7 shows the conditions to be met. Attention is called to the distance *BC*; it should be just beyond the focal distance of the transit. The focal distance is specified here because it will usually approximate the wire interval, *AB*. In a large shaft, or under circumstances where *AB* is much greater than 3.5 to 4.5 ft., the ratio *BC/AB* should be equal to or less than one. As a matter of fact, this should always be the objective, regardless of the wire spacing or the focal distance. When the angle *W* at *C* is but a few minutes, *AB* plus *BC* will equal *AC* within the limits of making the measurements. These two sets of measurements provide a check on each other. All distances are measured to the nearest thousandth of a foot. This is done to ensure accuracy to the nearest hundredth. In fact, an error of several hundredths of a foot in making the measurements will cause a discrepancy of only a few seconds in the results. This will be true if the Weisbach angle is small and *BC* and *AB* are similar in value. For example, let *AB* = 3.214 ft., *BC* = 5.121 ft., *AC* = 8.332 ft., and the measured angle = 0° 15' 10". Calling the angle at *A*, *x*,

$$x = (910'' \times 5.121) / 3.214 = 0^\circ\, 24'\, 10''$$

If an error was made in measuring *AB* (3.19) and *BC* (5.10), *x* will equal 0° 24' 15"; and if *AB* = 3.21 and *BC* = 5.10, *x* = 0° 24' 06"; and if *AB* = 3.23 and *BC* = 5.10, *x* = 0° 23' 57'.

With the exception of the last, they all fall within the nearest 10 sec. to which the known angle was turned. Certainly a system of measuring involving an error of 0.02 ft. in each distance looks crude. On the other hand, a slight motion of the wires might be responsible. If so, the results for ordinary work will not be seriously affected

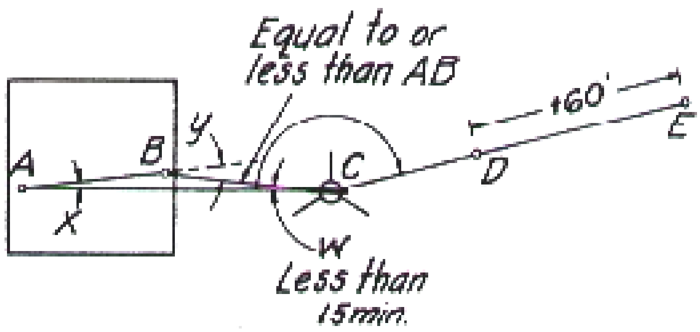


Fig 3.7 Shaft plumbing by triangulation (Weisbach) method

Returning to Fig. 3.7, the transit is set up as near on line as possible without resorting to coplaning. A string stretched along the wire plane and a point marked on the floor will prove helpful. If *CDE* cannot fall on line, *C* can be temporarily located with a lead block, as was done in coplaning. When *C* must be used as a backsight, *CD* should, if possible, be not less than 60 ft.

The safest procedure for turning the Weisbach angle is as follows. With the plate set at zero, backsight on the proper wire and turn the small included angle right. With a 1 min. transit, six repetitions, three of which are direct and three inverted, are probably all that are justified; and inversion of the telescope is essential to good work. Using the FS wire as a backsight, turn the large exterior angle right the same number of times. The sum of these angles should (if six repetitions were used) equal 360° ± 10". If it does not, and the wires were reasonably steady, the work should be repeated. On checking within the allowable limit, the two angles are adjusted by equally distributing the difference so that the sum will become 360°. This appears to be carrying the work to extremes; however, the object is to get a reliable azimuth to *DE*. When this is accomplished, one may usually forget about the seconds.

To continue the azimuth to *D*, take a backsight on one of the wires and turn the same number of angles as previously. Many engineers do this, using each wire as a backsight, and average the resulting azimuth; others, in addition, turn the exterior angle and adjust to 360 deg. Undoubtedly the same care should be exercised in establishing *E*. This will give an accurate azimuth for *DE*.

When very precise results are necessary, the procedure outlined should be repeated with the transit on both sides of the plumb plane and the two results averaged.

The angles at *A* and *B* (*x* and *y*, respectively) should both be found by calculating them from the measured data. A tendency exists to find *y* by difference [$y = 180 - (x + w)$]. This absolutely must not be done, since a check on the calculations is thereby lost, in addition to knowledge of a major discrepancy because of poor taping. The check is $x + w = y$. If difficulty is experienced in getting good measurements and turning *w*, *y* from *x* + *w* may not closely check *y* calculated. When this happens, the difference between the two *y*'s is applied equally to *x* and *y*. The turned angle *w* is not altered. This appears reasonable, since the value of *w is* independent of all linear measurements; but *x* and *y* depends on such measurements in addition to *w*. After correct values for *x* and *y* have been agreed upon, the bearing or azimuth of *AB* (which is known from the surface orientation) is readily transferred to *DE*.

Occasionally the shaft station layout readily adapts itself to establishing C as a permanent station, thereby eliminating *D*. At other times *DE* may be at right angles to *AB*, or C may be on the opposite side of the shaft from *DE*. Regardless of these ramifications of the problem, the triangulation method is as satisfactory as coplaning. Excessively short distances (exception *BC*) should be avoided as far as possible, as should too many intermediate stations, before arriving at the permanent base *DE*.

There is one mistake in particular that must be guarded against. This is basing the calculations of the azimuth with *C* on the opposite side of the plumb plane from its actual field position. If, when turning *w* by an angle right, the far wire occurs as the backsight, then *C* is on the left-hand side of the plumb plane when facing the wires. If the near wire is the backsight, *C* is on the right-hand side. This or some other rule must be followed in order to establish the location of *C*, and explicit notation to this effect must be entered in the notebook.

Example :

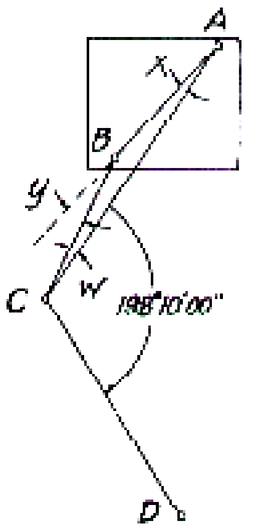


Figure 3.8 accompanies the following data:

bearing of *AB* = S 45° 26' 20"W,

length of *AB* = 4.235 ft.,

length of BC = 5.043 ft.,

length of AC = 9.280 ft.,

angle BCA = w = 0° 12' 40'

angle ACD = 198° 10' 00".

With this information the bearing of CD can be found.

The solution of the triangle ABC for angles x and y may be performed in one of two ways, both of which are presented here. The first set is the handiest, since it requires no trigonometric tables or log tables.

For small angles, the direct ratio of the angles to each other, without regard to any particular function (in this instance, the law of sines), gives sufficiently accurate results. The operation is identical with that of solving by the law of sines, except that that particular function is dropped, and the angle is simply converted to seconds.

w = 0° l2' 40" = 760",

$w : AB :: x : BC$,

760 : 4.235 :: x : 5.043,

x = 760 / 4.235 x 5.043 = 904.7" = 0º 15' 05"

Likewise,

760 : 4.235 :: y : 9.280,

y = 760 /4.235 x 9.280 = 1664.8" = 0° 27' 45".

Check:

$x + w = y$,

0° 15' 05" + 0° 12' 40" = 0° 27' 45".*

(* There is no adjustment to make. Had there been, it would be equally divided between x and y. For example, assume that $x + y$ = 0º 27' 55", and that the calculated x = 0° 15'15")

Then the difference is

0° 27' 55" - 0° 27' 45" = 10"

10 ÷ 2 = 5" correction to be applied.

0° 15' 15" + (180 - 0° 27' 45") + 0° 12' 40" =180º 00' 10".

The correction must be subtracted from x and added to y.

Corrected x = 0° 15' 15" - 0° 00' 05" = 0° 15' 10".

Corrected y = 0° 27' 45" + 0° 00' 05" = 0º 27' 50".

Check on correction = 180° = $x + (180 – y) + w$

= 0° 15' 10" + (180 - 0° 27' 50") + 0° 12' 40" = 180° 00' 00"

Using the law of sines to find *x* and *y* (this is the second of the two methods mentioned above; it requires a table of functions and logs),

sin *w* **:** *AB* **::** sin *x* **:** *BC*,

sin *x* = (sin *w* *BC*) / *AB*

sin *x* = (sin 0º 12' 40")(5.043) / (4.235) = 0º 15' 05",

sin *y* = (sin 0º 12' 40")(9.280) / (4.235) = 0º 27' 45".

These values check those first obtained.

Bearing of CD. To afford a check on the calculations, the bearing of *CD* should be found by following along both *ABCD* and *ACD*. Taking *ABCD* first;

Bearing *AB*	=	S 45° 26' 20" W		180° 00' 00"
		180	*y* =	0° 27' 45"
Azimuth	=	225 26 20	Angle *ABC* =	179° 32' 15"
Angle *ABC*	=	179 32 15		
		404 58 35		
		180		
Azimuth of *BC*	=	224 58 35		
Angle *w*	=	0 12 40		
Angle *ACD*	=	198 10 00		
		423 21 15		
		180		
Azimuth of *CD*	=	243 21 15	=	S 63° 21'15" W.

Proceeding by way of *ACD*,

Bearing *BA*	=	N 45° 26' 20" E	=	azimuth of *BA*
Angle *BAC*	=	359 44 55		360° 00' 00"
			x =	0° 15' 05"
		405 11 15		
		180	Angle *BAC* =	359° 44' 55"
Azimuth of *AC*	=	225 11 15		

Angle *ACD* = 198 10 00

423 21 15

180

Azimuth of *CD* = 243 21 15 = S 63° 21' 15" W.

The two results check each other.

If the coordinates of either or both wires are known, distance *CD*, vertical angles, HI and HS, the coordinates, and the elevation of *C* and *D* are at once obtained.

In conclusion, it is worth remarking that, by careful work, the azimuth of the lower end of the plumb wires should be ascertained within 15 sec. and the horizontal error of coordinates within 0.5 ft. per mile (this is of the order of 1 in 10,000).

For controlling the operations of many mines, such precision is not necessary. An error in the take-off azimuth of 1 min. or even as much as 5 min. would not prove serious. It may be stated that, as the shaft becomes deeper and the workings progress farther along the strike, correspondingly greater care must be taken with the shaft plumbing.

All of the foregoing discussion has been based on transferring the meridian down the opening. The same remarks and precautions hold equally true for running up a raise or shaft.

3.4.3. Four and Three Wire Methods

Figure 3.9 shows a method using four wires. On the surface or starting end, *A* and *E* are established and *B* and *D* are moved into line. Underground the transit is wiggled into position so that it is in both plumb planes. To facilitate this operation strings are stretched along *AB* and *ED* and the instrument is set up at their intersection.

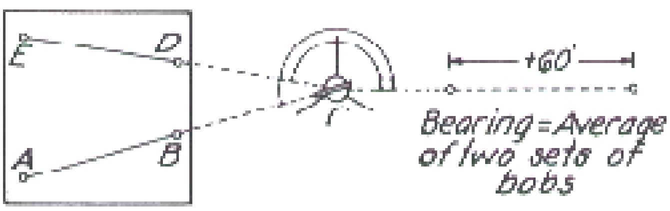


Fig 3.9 Four wire method

The transit is lined in at *a* set on zero, and sighted on w_2w_1 and a traverse is run to station 1 and station 2. After this is done, the transit is moved to *b* and the operation is repeated. The difference of the azimuth from the two traverses equals angle *D*.

The sum of the interior angles of a quadrilateral is equal to 360 degrees. In the quadrilateral w_1, *a*, station 1, *b*, w_1, only *C* is unknown. (It can, of course, be turned when station 1 is occupied.)

$C = 360 - (360 - A + B + D) = A - B - D.$

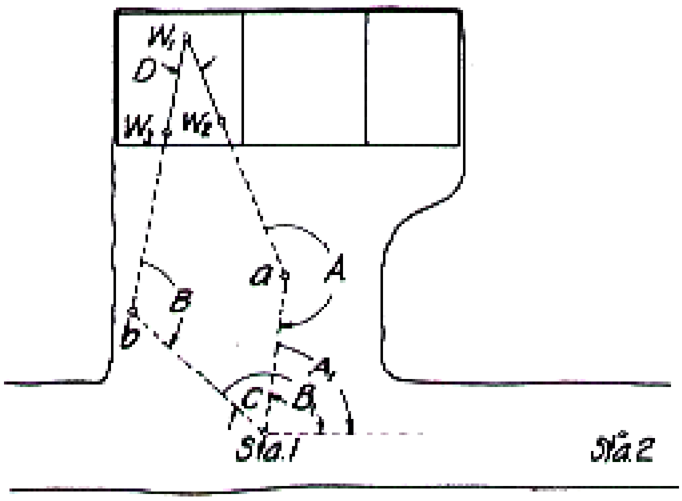


Fig 3.10 Three-Wire Method

Measuring *C* with the instrument provides a check on the other work, since *C* should be the same in each case.

When plumbing more than one level from the same wire setup, the angle *D* should be the same in each instance (that is, on each level where taken off).

3.4.4. Quadrilateral (Weiss) Method

Another method of using the quadrilateral is shown in Fig. 3.11 (known as the Weiss method). Using this figure permits a longer base line for *AB.* It may also prove desirable on certain shaft station layouts such as shown in the illustration. The surface bearing of *AB* is established by triangulation or coplaning. Underground C and *D* are put in as permanent or semipermanent stations. There is no need to have the sites parallel. The location of side *CD* is selected for convenience of instrumental work. Its length usually approximates that of *AB* and it ordinarily closely parallels *AB.* From C the angles *a, b,* and *e* are measured; at *D,* angles *c* and *d* are turned. With these five angles and the sides *CD* and *AB*, the bearing can be transferred to *CE.* There are three ways in which this may be done. One method does not make use of the length of *AB*. In the triangle *CBD* angle *i* is unknown. It is found from $180° - b - (c + d)$. From triangle *ACD,* $f = 180° - c - (a + b)$. Assume a bearing for *CD* (for convenience use the magnetic bearing). From the law of sines and *CD,* calculate *AC* in triangle *ACD* and *CB* in triangle *CBD.* On the basis of the assumed bearing for *CD,* the angles given, and the calculated sides, the coordinates of *A* and *B* are found. From these sets of coordinates the bearing of *AB* is determined. A comparison of this bearing with the true bearing of *AB* gives the correction to be applied to the assumed bearing of *CD.*

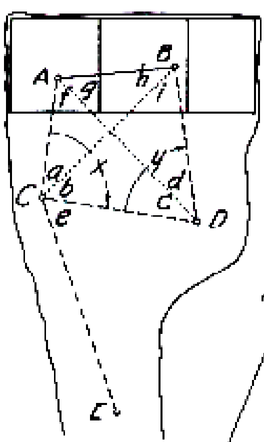


Fig 3.11 Weiss method

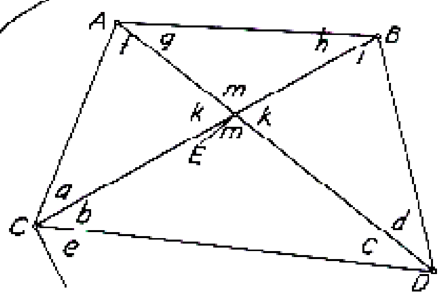


Fig 3.12 Weiss quadrilateral

For the second procedure *AB* must be used. The object here is, by applying the law of sines, to find the angles *g* and *h.* Thus the bearing of *AB* may be directly transmitted to *CE* without using an assumed bearing for *CD. So* far as the actual labor of computation is concerned, there is little difference between the two; if anything, the latter method is a little shorter, whereas the former has the distinct advantage of not including the uncertainty involved in measuring *AB.* Angles *f* and *i* are found as before. By simple proportion and the law of sines, it is found that

$$\sin h = \frac{\sin a \sin c}{\sin f}\frac{CD}{AB} ,$$

$$\sin g = \frac{\sin d \sin b}{\sin i}\frac{CD}{AB} .$$

By a similar use of their functions *g* and *h* may be found without using *AB.* Figure 3.12 shows the setup. Here

$$\text{angle } k = 180° - f - a = 180° - d - I,$$

$$\text{angle } m = 180° - b - c = 180° - g - h,$$

$$AE = \frac{\sin a}{\sin k}\frac{\sin c}{\sin f} CD ,$$

$$BE \quad = \quad \frac{\sin d}{\sin k}\,\frac{\sin b}{\sin i}\,CD\,.$$

From trigonometry, having two sides and the included angle (in our instance, *AE*, *BE*, and *m)*,

$$½\,(g + h) \;=\; 90^{\circ} \;-\; ½\,m,$$

$$\tan ½\,(g - h) \;=\; \frac{BE\,-\,AE}{BE\,+\,AE}\,\tan\frac{1}{2}\,(g+h)\,,$$

from which

$$g \;=\; ½\,(g + h) \;+\; ½\,(g - h),$$

$$h \;=\; ½\,(g + h) \;-\; ½\,(g - h).$$

As a check on the measured distance of *AB* and incidentally on the determination of *g* and *h*, *AB* may be found from

$$AB \quad = \quad \sqrt{BE^{2}\,+\,AE^{2}\,-\,2BE\;\;AE\cos m}\,.$$

The results to be expected here would depend on the procedure for finding the center of swing for the wires.

3.5. Two-Shaft Method

Generally the method of hanging one wire in each of two shafts or raises and running a traverse between them gives the most reliable results and should be used at every opportunity. For most control work the one-shaft methods of coplaning or triangulation are entirely satisfactory. In some types of work (subways, for instance) very precise alignment and closure are desirable. This may also be necessary in some deep and extensive mines. If two vertical openings, connected at their bottoms by drifts (horizontal or inclined), are available, then the two-shaft method can be used. In many instances, especially relatively shallow workings, churn-drill holes are sunk in order to provide the two openings. The wires are hung in the shaft and the drill hole. All of the precautions so far discussed for hanging and quieting the bobs must be observed.

The procedure consists of tying the upper ends of the wires together by means of a traverse and then calculating the horizontal distance and bearing between the wires. In running the traverse the method used depends on the precision desired. For the usual mine work (1 in 10,000) no attention need be paid to temperature and tension. When a triangulation system is present, the wires may be tied together by that system. For the underground traverse the transit is set up near one of the wires (about 60 ft. from it, if possible), and a backsight is taken on the wire and the distance measured. The bearing from the wire to the instrument is assumed. Any value may be taken; less confusion results if the magnetic bearing is used, as it usually will closely approximate the true bearing. A traverse is then run to the other wire. With these data the distance between the wires and the bearing is calculated. The underground distance should check the surface distance within the limits set for the work. If it does not, the source of error must be discovered and corrected or distributed. Ordinarily, a traverse is run each way (both surface and underground), using a different set of stations on each check route. In this way a check in each instance is obtained and the allowable closure distributed.

The amount by which the bearing of the wires differs from the surface bearing is the correction to be applied to the assumed bearing. The new coordinates of the underground survey are then calculated and the survey is correctly oriented.

Figure 3.13 shows diagrammatically the method. The surface traverse is run from wire *x* to wire *y* by stations 1, 2, 3, etc. A return check traverse is run, but not along the identical route of the first. If the distance *xy* checks within the limit prescribed, their average represents the surface distance. The difference between the north coordinates of *x* and *y* and the east coordinates of *x* and *y* gives two legs of a right triangle. From these distances, by using the tangent, the bearing of *xy* is found.

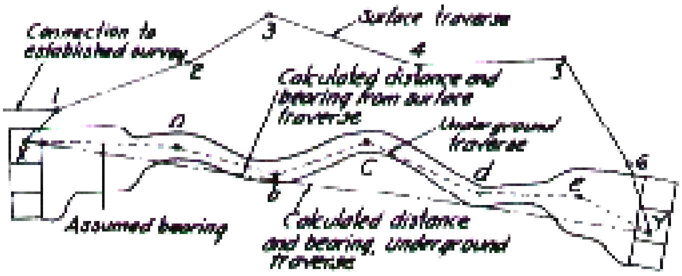


Fig 3.13 Two-shaft method of transferring the meridian.

Going underground, the traverse is started at *a*. With the instrument at *a*, a backsight is taken on *x* and the traverse run to *y* through *a, b, c,* etc. A return check is run, following the same stipulation as given for the surface survey. The magnetic bearing *x* to *a* is assumed for calculating the underground work. (The needle reading on the transit from *a* to *x* is recorded; reversing its quadrant gives *x* to *a.)* The underground bearing of *xy* is calculated exactly as for the surface bearing. A comparison of the two gives the correction to be applied to *xa.* Recalculating the underground survey with the correct bearing of *xa* orients this traverse with respect to the surface work; for the starting coordinates use the same values as *x* at the surface. Both traverses are run and checked prior to hanging the wires.

Example :

Figure 3.14 is a portion of an underground and surface survey. The underground survey is represented by letters. The plumb wires are at *x* and *y.* Data necessary for calculating the survey are given in Table 3.2. To simplify the illustration, angles to the nearest 15 min. are recorded and distances to the nearest 0.5 ft. In practice the least reading of these measurements would be *at least* 1 min. and 0.01 ft. Also the field data would include vertical angles and slope distances instead of horizontal distances.

Solution:

The bearings are calculated first by the method already discussed. They are recorded in the table. Next the latitudes and departures are found and then the coordinates. These results are given in Table 3.2

Surface: Distance *xy* and bearing.

	N		E
x	9161.18	*x*	10,846.14
y	9145.29	*y*	10,659.93
Lat.	15.89	Dep.	186.21

$$xy = \sqrt{15.89^2 + 186.21^2} = 186.89 \text{ ft.,}$$

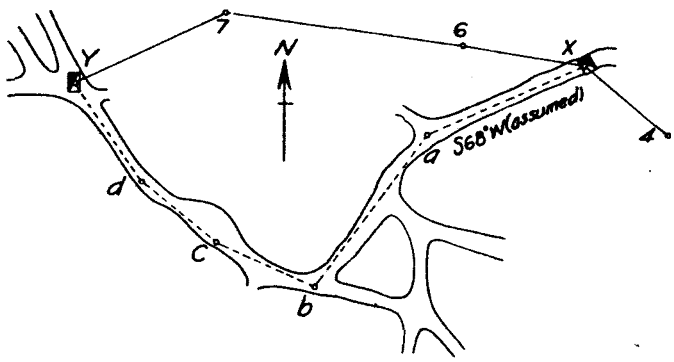


Fig 3.14 Example of two-shaft method.

bearing *xy* = $\tan^{-1}$ = dep / lat = 186.21 / 15.89 = S 85° 07' 20" W

Underground : Distance *xy* and bearing.

	N		E
X	9161.18	*x*	10,846.14
Y	9149.59	*y*	10,659.58
Lat.	11.59	Dep.	186.56

$$xy = \sqrt{11.59^2 + 186.56^2} = 186.92 \text{ ft.}$$

Correction of assumed bearing: Take the underground section of the traverse,*yxa*. Find the angle right at *x*, *y* backsight, *a* foresight. (The bearing found for *xy* must be reversed to give the bearing of *yx*, and is N 86° 26' 40" E.) As was discussed under bearings, the angle right between two courses is the azimuth of the FS course + 180° - azimuth of the BS course. Angle right at *x* = (180° + 68°) + 180° - 86° 26' 40" = 341° 33' 20". Attention is called to the fact that the underground angle right at *x*, with *y* as a backsight and *a* as the foresight, will have the same value regardless of the value assumed for *xa*. Therefore, taking the surface bearing of *yx*, which is the correct bearing, and adding this angle right will give the true azimuth of *xa*.

(N 85° 07' 20" E + 341° 33' 20") - 180º = 246° 40' 40".

Or the bearing is

246° 40' 40" - 180° = S 66° 40' 40" W.

The correction to *xa* assumed is

68° 00' 00" - 66° 40' 40" = 1 °19' 20".

There are other ways of finding this correction. The one given is the least confusing and, besides, actually represents what takes place.

The underground survey is corrected by 1° 19' 20" and recalculated for plotting.

BS	IS	Angle right	HD	Bearing	Latitude		Departure		Coordinates		FS
					N	S	E	W	N	E	
2	3	-		-					9101.00	10,926.00	4
3	4	-	100.00	N 53º W	60.18			79.86	9161.18	10,846.14	x
4	x	150º 00'	45.00	N 83º W	5.48			44.66	9166.66	10,801.48	6
x	6	178 30	90.00	N 84º 30' W	8.63			89.59	9175.29	10,711.89	7
6	7	144 30	60.00	S 60º W		30.00		51.96	9145.29	10,659.93	y
-	x	-	62.50	S 68º W (assumed)		23.41		57.95	9137.77	10,788.19	a
x	a	146 30	70.50	S 34º 30' W		58.10		39.93	9079.67	10,748.26	b
a	b	261 45	42.00	N 63º 45' W	18.58			37.67	9098.25	10,710.59	c
b	c	191 00	39.50	N 35º 30' W	23.91			31.44	9122.16	10,679.15	d
c	d	197 15	33.70	S85º07'20"W	27.43			19.57	9149.59	10,659.58	y
	x_s		186.89	S86º26'40"W							y_s
	x_y		186.92	S66º40'40"W							y_u
	x_c										a_c

Note : x_s , y_s equal the surface end of the wires.
X_u, y_u equal the underground end of the wires.
x_c, a_c equals corrected bearing of xa.